The results of simulation of the process of the disposal (post-mission disposal or disposal after
failure as a result of emergency) of a spacecraft in quasi-geostationary orbit are given. The necessary
impulses, fuel consumption for maneuvers, the required time for the raising the orbit to 300 km,
changes in the major semi-axis and eccentricity of the orbit and the satellite’s drift in longitude are
calculated. The process of raising the orbit is considered as a result of successive starts of gas engines
(GE) and corresponding changes in the indicated orbital parameters. The dependence of the satellite
drift and longitude for different values of eccentricity is presented. It is shown that, within the maximum
duration of the engine operation time of the GE, the duration of their individual starts does not
significantly affect the final results of the disposal.

Keywords: Spacecraft; geostationary orbit; disposal; maneuvers; numerical simulations; orbit
parameters

Introduction

A geostationary satellite whose service life ends, in accordance with the recommendation of
the International Telecommunication Union [1], should be withdrawn from the geostationary orbit
(GSO) area until the fuel stocks are exhausted in order to avoid the risk of collision with operating
GSO satellites. At the same time, the minimum perigee height should ensure that the satellite does
not return to the protected GSO zone after disposal from the geostationary orbit, which extends 200
km above the GSO. For satellites with orbital eccentricity $e < 0.003$, the required altitude is defined
as:

$$\Delta H > 235 + 1000 \frac{Cr A}{M},$$

where $Cr$ is the coefficient of reflectivity at the beginning of the service life, $A$ is the area of
the satellite angle exposed to the Sun, $M$ is the dry mass of the satellite.

The Control Center of these Spacecrafts (SC) should monitor the fuel consumption on board to
ensure that the amount necessary for maneuvering to take off from the GSO is available. In
addition, it is required to have a fuel stock to take into account the effect of possible inaccuracies in
determining the orbit and the errors of maneuvering.

The results of work [2] show that geosynchronous orbiting satellites at the end of their service
life should be brought to a height of not less than 300 km above the GSO.

Thus, the disposal of spacecraft from a geostationary orbit is regulated by international legal
acts. Although they are recommendatory in nature, in fact, they are mandatory for implementation.

Since the issue of flight safety of satellites is always one of the most pressing issues, a lot of
work is devoted to this issue. For example, it was shown in [3] that the required perigee altitude of
the burial orbit cannot exceed 300 km from the height of the upper boundary of the protected region
of the GSO. In addition, estimates of the necessary specific impulses of the control force for
performing escape maneuvers consisting of a change in the major semi-axis of 500 km have been
obtained. In particular, a specific impulse of about 18 m/s for a period of not more than one year is
required for disposal from GSO.
An analysis of the fullness of GSO by spacecrafts, statistics of the disposal from orbit for 1997-2004 on the basis of TLE-elements was carried out in [4]. It is shown that at present, some of the satellites abandoned in GSO represent a danger to operating satellites. This once again proves the necessity of maneuvering the drift for each GSO satellite at the end of the active life or after failure as a result of an abnormal situation.

In [5-10] also problems related to the features of the disposal from GSO of satellites, including satellites possessing a non-zero, even significant inclination angle, are also considered. Taking into account the urgency of the problem of the SC disposal from GSO, the present work considers a method for modeling this process, for estimating the important parameters of such a maneuver: the necessary impulse, fuel consumption, required time, longitude drift and eccentricity changes. Since in most cases SCs are in a quasi-geostationary orbit with a small eccentricity and inclination angle, here we confine ourselves to this case. The case of moving a spacecraft from an ideal geostationary orbit (e = 0) was considered in [11].

1. Controlling the motion of the GSO satellite

Currently, geostationary satellite motion can be controlled using large and small thrust engines [12, 13]. In the case of high-thrust engines, it is assumed that the time it takes to create the necessary speed increment $\Delta V$ is negligible compared to the satellite revolution period. This kind of control is called impulse control. In this case, the action of the traction force is reduced to a sudden change in the velocity of the spacecraft without changing the coordinates during the time the engine is running [13]. Given the fact that this time is usually much shorter than the time of the orbital transition, this assumption is justified.

In the case of low-thrust engines, the running time of the engine becomes of the order of the satellite revolution period. In this case, the impact of the traction force on the orbit can be calculated by numerical integration of the differential equation of motion of the spacecraft, the acceleration due to the thrust of the engine [12, 14] on the right-hand sides of which is included. On this basis, in [15], the problem of controlling the planar parameters of the orbit of a geostationary spacecraft with the help of a low-thrust engine is solved. Since all maneuvers must be carried out from the optimality condition, in [16] the problem of optimizing the geostationary orbit with the use of an ionic low-thrust engine is considered.

As is known, if the inclusion of the engine can be considered impulse, then the velocity increment can be estimated as follows [11]:

$$\Delta V = \int_{t_0-\Delta t/2}^{t_0+\Delta t/2} \frac{F}{m} \, dt \pm \frac{F \cdot \cos \alpha}{m} \Delta t$$

(1)

where $F$ is the vector of thrust, $m$ is the mass of the space vehicle, $\alpha$ is the angle between the directions of maneuvering and thrust of the engine, $t_0$ is the calculated moment of the impulse, and $\Delta t$ is the engine running time. In this case, the mass of the spacecraft can be considered constant during the operation of the engine. Since, in fact, the mass of the spacecraft is a function of time, if necessary, as the mass of the space vehicle, its mean value over the initial and final values during maneuver can be taken.

Of the three kinds of impulses (tangential, normal and lateral, or binormal), a tangential impulse is used to change the major semi-axis of the orbit, the drift speed in longitude and the eccentricity vector. Taking into account that in the problem considered by us, as a result of one small impulse ($\Delta V \ll V$), the major semi-axis changes by a small amount ($\Delta a \ll a$), it can be shown that [12-14]:

$$\Delta a = \frac{2a^2 \Delta V}{\mu} \Delta V,$$

(2)
where \( \mu = fM \) is the gravitational parameter of the Earth.

The change of the major semi-axis is simultaneously accompanied by a change of the eccentricity \( e \) [12]:

\[
\Delta e = \frac{2(e + \cos \nu)}{V} \Delta V ,
\]

(3)

where \( \nu \) is the true anomaly of the spacecraft.

Another parameter of the movement, which will change as a result of the change in the major semi-axis, is the satellite drift speed on the GSO. In the case of \( e \neq 0 \) and \( i = 0 \) (inclination of the orbit), the speed of drift is defined as [12]:

\[
\dot{\lambda} = 2\pi \left( \frac{1 + 2e \cdot \cos \nu}{T_s} - \frac{1}{T_E} \right) ,
\]

(4)

where \( T_s \) is the sidereal period of the satellite revolution, \( T_E \) is the duration of the stellar day.

The difference of the sidereal period from the stellar day will lead to the appearance of a drift along the longitude. In this case, the longitude of the spacecraft at time \( t \) is defined as (in the linear approximation):

\[
\dot{\lambda} = \lambda_0 + \dot{\lambda}(t-t_0) ,
\]

(5)

where \( \lambda_0 \) is the longitude of the spacecraft before the start of the drift at the time \( t_0 \).

The use of (4) allows us to take into account not only the evolution of the mean longitude, but also the longitude variations around the mean value, caused by the presence of eccentricity and natural short-period variations.

If in the process of transferring the spacecraft from one point of standing to another or when putting a burial place into an orbit, the eccentricity is not superimposed, then by means of a tangential impulse, a transferring can be started at any time.

### 2. Results of modeling and discussions

As in [11], we consider the process of moving a spacecraft from a GSO to a height of 300 km. The process of rising of the orbit will be considered as a result of successive starts of gas engines (GD) and corresponding changes in the indicated orbital parameters.

To simulate the disposal process, the following initial parameters have been adopted:
- the major semi-axis of the orbit is \( a_0 = 42164125 \) m;
- eccentricity \( e_0 = 0 \);
- circular speed \( V_0 = 3075 \) m/s;
- mass of space vehicle \( m_0 = 1080 \) kg;
- thrust of the engine \( F = 0.009 \) N;
- angle between the directions of maneuvering and engine thrust \( \alpha = 60^\circ \);
- mass fuel consumption for one gas engine = 0.016 g/s;
- number of simultaneously operating gas engines \( N = 4 \);
- initial longitude of spacecraft \( \lambda_0 = 103^\circ \);
- engine operating time \( \Delta t = 1800 \) s; 5000s.

Numerical simulation is performed in the following sequence.

1) Calculation of the increment of the velocity \( \Delta V \) by the formula (1). In this case, for each individual maneuver, as spacecraft mass is taken its mean value \( m_m = (m_{i-1} + m_i) / 2 \), where \( m_{i-1} \) and \( m_i \) are the masses of the spacecraft at the beginning and end of the maneuver. The spacecraft mass at the end of the maneuver is defined as \( m_i = m_{i-1} - dm = m_{i-1} - m \cdot \Delta t \).
2) Calculation of the change in the major semi-axis $\Delta a$ by formula (2). At the same time, as the major semi-axis $a$ and velocity $V$, values are taken that correspond to the beginning of the maneuver.

3) Calculation of the change in eccentricity $\Delta e$ by formula (3). As in the previous case, as the velocity $V$, values are taken that correspond to the beginning of the maneuver. After the maneuver, the velocity, the major semi-axis and the eccentricity will become, respectively

\[ V = V_0 + \Delta V, \quad a = a_0 + \Delta a, \quad e = e_0 + \Delta e. \]

4) Calculation the sidereal period of revolution of the satellite $T_s$ corresponding to the changed major semi-axis $a$, and the duration of the stellar day $T_E$ corresponding to the major semi-axis of the ideal orbit $a_0$ by the formula:

\[ T = 2\pi \sqrt{\frac{a^3}{\mu}}. \]

5) Solution of the Kepler equation and finding the eccentric anomaly $E$, where $M$ is the mean anomaly:

\[ M = E - e\sin E. \]

6) Calculation of the true anomaly $v$:

\[ \tan \frac{v}{2} = \sqrt{\frac{1+e}{1-e}} \cdot \tan \frac{E}{2}. \]

7) Calculation of the speed of the drift $\dot{\lambda}$ by the formula (4).

8) Calculation of the longitude value of the spacecraft at the end of the maneuver:

\[ \lambda = \lambda_0 + \dot{\lambda} \cdot \Delta t. \]

In order to determine the effect of the duration of the engine on simulation results, in [11] the calculations were carried out at $\Delta t = 1800$ s, 3600 s and 5000 s. The last value is chosen based on the limitations on the duration of operation of some gas engines that are used in practice. Calculations showed that in all three cases the results are almost identical.

Since the case $e=0$ was considered in [11], in the present paper the effects on the results of modeling not only the duration of the engine operation, but also the eccentricity of the orbit are studied. For this purpose, we have considered the cases $\Delta t = 1800$ s and $\Delta t = 5000$ s with different eccentricity values (Table 1).

<table>
<thead>
<tr>
<th>$e$</th>
<th>$\Delta t$, s</th>
<th>Nesc. time, day</th>
<th>$\Delta V$, m/s</th>
<th>Fuel consumption, kg</th>
<th>$\Delta e$</th>
<th>$\Delta \lambda$, degree</th>
<th>$\dot{\lambda}$, degree</th>
</tr>
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<tbody>
<tr>
<td>0.01</td>
<td>1800</td>
<td>7.3851</td>
<td>10.841</td>
<td>40.837</td>
<td>1.52e-04</td>
<td>13.418</td>
<td>89.582</td>
</tr>
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<td></td>
<td>5000</td>
<td>7.3854</td>
<td>10.841</td>
<td>40.838</td>
<td>1.75e-04</td>
<td>13.268</td>
<td>89.732</td>
</tr>
<tr>
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<td>1800</td>
<td>7.3851</td>
<td>10.841</td>
<td>40.837</td>
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<td>87.569</td>
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<td>10.841</td>
<td>40.838</td>
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<td>14.926</td>
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<td>72.009</td>
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</table>
As can be seen from this table, the required time for rising of the orbit to 300 km, the necessary increment of the velocity $\Delta V$ and fuel consumption are practically independent of the eccentricity and the duration of the engine operation. In this case, the eccentricity of the orbit remains practically unchanged throughout the entire process of disposal. But the value of the eccentricity significantly influences the drift speed and the finite value of the longitude of the spacecraft.

The calculations showed a linear dependence of the change in the major semi-axis on time, and $\Delta a = 300$ km is achieved in 7.385 days. Figure 1 shows the longitude drift $\dot{\lambda}$ as a function of time for the two extreme values of eccentricity considered. As can be seen from the figure, the use of this expression for $\dot{\lambda}$ made it possible to reveal not only the evolution of the mean drift, but also its oscillations around the mean value, caused by the presence of eccentricity.

![Fig. 1. Changes in the drift of the longitude ($\dot{\lambda}$) with eccentricity values e = 0.01 (a) and e = 0.07 (b).](image)

Figure 2 shows the changes in longitude ($\lambda$) also with the two extreme values of eccentricity considered. Here, as well as in figure-1, not only the evolution of mean longitude, but also its oscillations around the mean value, caused by the presence of eccentricity and natural short-period variations, are revealed.

In both cases, an increase in the eccentricity resulted in an increase in the amplitude of the oscillations of the parameters considered, drift and longitude of the space vehicle.

On the basis of the results obtained, it can be argued that, when a spacecraft is taken from a GSO with permanently-powered engines, the eccentricity does not affect the fuel consumption and rise time of the orbit to the disposal orbit. But it has a significant influence on the drift and longitude change of spacecraft.

In order to test the need for such a simulation, we will consider the process of moving a spacecraft from a GSO as one whole maneuver.

Calculation of the velocity increment $\Delta V$ by the formula $\Delta V = \mu \cdot \Delta a / 2a^2V$, where $\Delta a = 300$ km, $V=V_0=3075$ m/s and $a=a_0=42164125$ m gives $\Delta V=10.94$ m/s. This value differs from the value of $\Delta V$ obtained as a result of modeling by less than 1%. This gives grounds to state that in order to estimate the necessary velocity increment $\Delta V$ to disposal spacecraft from GSO, there is no need for detailed simulation of the process.
Similarly, calculating the change in eccentricity using the formula \( \Delta e = 2 \cdot \Delta V / V \), where \( \Delta V = 10.94 \text{ m/s} \) and \( V = V_0 = 3075 \text{ m/s} \) also gives a very close values to the simulation results: \( \Delta e = 0.0071 \), which is a deviation of 1.43%.

Unlike the previous parameters, a significant difference arises in the estimation of the operating time of the engines: \( \Delta t = \Delta V \cdot m / F \cdot \cos \alpha = 2625600 \text{ s} \). Taking into account the fact that 4 engines operate simultaneously: \( \Delta t = 656400 \text{ s} = 7.5972 \text{ days} \), i.e. the deviation is 0.2118 days.

As a consequence of the longer engine operation, there will be higher fuel consumption: 42.010 kg for the entire maneuvering period, i.e. 1.172 kg more than in the case of modeling.

The above parameters determine the longitude drift of the spacecraft. After moving the spacecraft to a height of 300 km above the GSO, the change in the period of revolution is \( \Delta T = 921 \text{ s} \). This in turn leads to drift = 3.8382 degrees per turn. Taking into account the disposal (moving) time at 7.5972 days, we get the longitude displacement \( \Delta \lambda = 29.16 \text{ deg.} \), which differs from the simulated value by 14.979 deg. Thus, the spacecraft had to be at longitude \( \lambda = 73.84 \text{ deg.} \)

A tangible difference in the estimates of the duration of the engine operation is associated with taking into account (or not taking into account) changes in the mass of the spacecraft due to fuel consumption. As a result of this, there are differences in the estimates of the total required fuel mass and changes in the longitude of the space vehicle.

Thus, it can be argued that a more accurate simulation of the SC disposal process from the GSO is justified in critical cases associated with the fuel stock, the time reserve for disposal or drift by longitude. This will allow you to choose the best variant for this maneuver.

**Conclusion**

Numerical simulation of the process of spacecraft disposal from a quasi-geostationary orbit to a height of 300 km has been carried out for the example of a spacecraft with a mass of 1080 kg, initial longitude 103°, equipped with gas engines. The changes in the major semi-axis and the eccentricity of the orbit, the longitude drift of the satellite, and the fuel consumption is considered as a result of consecutive starts of gas engines.

Simulation was carried out for three values of the duration of the engine: \( \Delta t = 1800 \text{ s}; 3600 \text{ s}; 5000 \text{ s} \). Calculations have shown that, within the maximum duration of the operations of gas engines, the duration of their individual operations does not significantly affect the final results of the disposal from orbit.
To assess the need for such a simulation, the process of moving SC from GSO is also considered as a one-time maneuver.

A tangible difference in the estimates of the duration of engine operation is due to the change in the mass of the spacecraft due to fuel consumption. As a result of this, there are differences in the estimates of the total required fuel mass and changes in the longitude of the space vehicle.

The analysis allows us to state that a more accurate simulation of the process of disposal of a spacecraft from a geostationary orbit is justified in critical cases associated with a fuel reserve, a time reserve for disposal or with a restriction on longitude drift.

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