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INFORMATION-ENTROPY ANALYSES OF THE QUALITY OF MANUFACTURING PROCESS OF TECHNOLOGICAL PRODUCTS

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Perfection of manufacture of copper considering the growing poor and complex-structured raw material is impossible only on a basis of traditional methods of opening the relationships of cause and effect during the general technological circuit. These methods require additional probable aspects which are taking into account casual character of the valid transformations of substance during a course of realization of the technological circuit with relation to both the basic product, and to accompanying valuable or harmful impurity.

Keywords: hierarchical system, accumulation, entropy, technological repartition, stochasticity.

For a multilevel hierarchical system of technological repartition it is important to describe a subordinate level as interaction of the interconnected subsystems each of which possesses the information properties. Therefore at reception of information estimation the basic attention is inverted on introlevel and interlevel interactions. The considered approach, in our opinion, fully complies with the basic requirements of the systematic entropy-information analysis as while modeling hierarchical system of technological processes it provides integrity of its consideration due to the general-theoretical and methodical concepts allowing entirely keeping in sight all system as a whole for the decision of a task at all levels. Besides, on the basis of the account of basic elements in system and connections between them it provides complete and multifold consideration. The suggested algorithm of simplification at modeling allows to reflect real technological repartition adequately and to take into account determining factors in hierarchical system.

Quantitative estimations of sense and value of the information can be made for the information analysis of quality of technological products and processes of their reception only after the preliminary agreement about what precisely in each concrete case has value and sense for the considered phenomena. Methods of calculation the information suggested by Shannon allow to reveal a ratio of quantity of the predicted information and quantities of the unexpected information which cannot be predicted beforehand, and thus to enable to define a qualitative and quantitative estimation of the certain technological circuit. As a probability of detection of the main element of technological system it is possible to accept its maintenance in a product, expressed in shares of unit. For example, let’s examine the maintenance of a considered chemical element, in our case - copper, in products of technological repartition. Also for probability of detection it is possible to take the maintenance of suitable fraction (remnants, briquettes) in a corresponding product. The same concerns the process of extraction of an element in this or that product, as in this case a parameter of extraction is equal to a probability of transition of the given element from one condition of system into another. These both parameters - the maintenance and extraction - can be equally used for an estimation of quality of a product or technological repartitions.

Let's show how quality of technological products and the technological processes resulting in reception of these products is estimated by results of technological repartitions copper-liquating manufactures on Zhezkazgan and Balkhash copper-smelting plants (CSP) (table 1). So, the maintenance of copper in ore makes 0,5-1,2% (on the average 0,85%), and in concentrates 5,5-40% (on the average 22,75%). Stein of melting in a liquid bath contains 40-55% copper (on the average 47,5%). The basic result of the work carried out on a scientific, technological and technical substantiation of process of converting finally is reduced to an opportunity to increase the maintenance of copper in draft metal. This parameter varies within the limits of 98,6-98,9% (on the average 98,75%). As a result of technological process of anodic melting the parameters of the maintenance of copper in anodes are the following 99,2-99,5% (on the average 99,35%). In process of electrolyte refinements parameters of the maintenance of copper in cathodes make 99,9-99,99% (on the average 99,95%).

Table 1. The maintenance of copper in products on Zhezkazgan and Balkhash CSP

Repartition	The name	Maintenance	Average value
Extraction	Ore	0,5-1,2%	0,85%
Enrichment	A concentrate	5,5-40%	22,75%
Melting	Stein	40-55%	47,5%
Converting	Draft copper	98,6-98,9%	98,75%
Fire refinement	Anodic copper	99,2-99,5%	99,35%
Electrolyte refinement	Cathode copper	99,9-99,99%	99,95%

For accounting of a various degree of unexpectedness (probability) of events C.Shannon has suggested to use probabilities' function of entropy borrowed from statistical physics, resulted as [1]:

$$H = -\sum_{i=1}^{N} p_i \log_2 p_i , \qquad (1)$$

where p_i – is a probability of detection of any homogeneous element of system in their set N,

$$\sum_{i=1}^{N} p_i = 1,\ p_i \geq 0,\ i = 1,2,\ldots,N .$$

The mathematical description of development of any system is set by the formula:

$$\frac{\partial^2 I}{\partial M^2} > 0, \qquad \frac{\partial^2 I}{\partial N^2} > 0,$$

where M - weight of technological system; N - number of elements of technological system.

The positive second derivative testifies the accelerated development of the system. The essence of this acceleration is that at transition to a higher structural level of technological process the law or a principle of progressive increase of variety comes into effect [2]. In mathematical understanding the principle of increase of variety means the following: with transition to higher structural levels the number of the elements forming the given structural level, having various attributes, increases under the law:

$$N_n = N_0^{k^n}, \qquad (2)$$

where $N_n = N_0^{k^n}$, n - number of levels, k - length of a code of elements at each level of hierarchical system.

Before the publication of K. Shannon's theory R.Hartly has suggested to define quantity of the information under the formula [3]:

$$H_{n(\max)} = \log N_n = \log N_0^{k^n} = k^n \log N_0 , \qquad (3)$$

where $N_n = N_0^{k^n}$, n - number of levels, k - length of a code of elements at each level of hierarchical system.

The theorem 1 *Let* N_n *- number of elements of* n*- level.* I_0*- capacity of the information of a zero level of technological system. Then the capacity of the information of* n*-level counting upon one element is expressed by the formula:*

$$I_n = k^n I_0 .$$

The theorem 2 *Let accumulation of the information at each level of technological repartition occurs without delay (acceleration) that is it is expressed by equality:*

$$\frac{\partial a_n}{\partial N} = \frac{\partial^{n+2} I}{\partial N^{n+2}} = 0 . \qquad (4)$$

Then the general rate of accumulation of the information at each level and the total of the information is the sum of the information inherent in each level of technological repartition are defined by:

$$\frac{I_n}{a} = \frac{N^{n+1}}{(n+1)!}, \qquad \frac{I_{\sum_n}}{a} = \sum_{i=0}^{n} \frac{N^{i+1}}{(i+1)!},$$

where a *- speed of accumulation of the information, a constant at all levels,* n *- number of levels of technological system,* N *- number of elements of technological system.*

Let's estimate a residual member of Mac Loren's line for function $e^N - 1$. No matter what was N, the residual member satisfies to a condition $R_n = \frac{N^{n+1}}{(n+1)!} e^{\theta N} \to 0$ at $n \to \infty$.

Really, as $0 < \theta < 1$, the size $e^{\theta N}$ at fixed N is limited. We shall prove, that what fixed number N was, the condition will be executed $\frac{N^{n+1}}{(n+1)!} \to 0$ at $n \to \infty$. If N - the fixed number there will be such positive number K that the inequality will be executed $|N| < K$.

Let's enter a designation $\frac{|N|}{K} = q$ then as q satisfies to an inequality $0 < q < 1$, we can write:

$$\left| \frac{N^{n+1}}{(n+1)!} \right| = \left| \frac{N}{1} \cdot \frac{N}{2} \cdot \ldots \cdot \frac{N}{n} \cdot \frac{N}{n+1} \right| < \frac{N}{1} \cdot \frac{N}{2} \cdot \ldots \cdot \frac{N}{K-1} \cdot q \cdot q \cdot \ldots \cdot q = \frac{N^{K-1}}{(K-1)!} q^{n-K+2} .$$

The size $\frac{N^{K-1}}{(K-1)!}$ is a constant, that is it does not depend from n, and size $q^{n-K+2} \to 0$ at $n \to \infty$. Therefore it is fair $\lim_{n \to \infty} \frac{N^{n+1}}{(n+1)!} = 0$. Hence $R_n = \frac{N^{n+1}}{(n+1)!} e^{\theta N} \to 0$ at $n \to \infty$.

From here follows, that at any N, having taken sufficient number of members, we can calculate size $\frac{I_{\sum_n}}{a}$ with any degree of accuracy. Thus, function $\frac{I_{\sum_n}}{a}$ has a limit; character of dependence of this limit from N is exponential.

The theorem 3 *Information capacity of technological system depends on information properties of system at various levels, a principle of development of system and is defined by:*

$$I_n = \frac{\Delta I_n}{(n+1)!},$$

where ΔI_n *- the maximal increment of the information.*

Under ΔI_n the maximal increment of the information is meant. As was already earlier shown the maximal increment of the information ΔI_n of technological system is distributed between the determined $I_n(d)$ and stochastic $I_n(h)$ parts in such a manner that equality is executed:

$$\Delta I_n = I_n(d) + I_n(h). \tag{5}$$

The theorem 4 *Information capacity of hierarchical system and n-level are defined by:*

$$I_{\sum_n} = \sum_{i=0}^{n} \frac{H_{i(\max)}}{(i+1)!} = \log N \sum_{i=0}^{n} \frac{\prod_{m=0}^{i} k_m}{(i+1)!}, \qquad I_n = \frac{H_{n(\max)}}{(n+1)!} = \frac{\prod_{m=0}^{n} k_m \log N}{(n+1)!}, \tag{6}$$

where $H_{n(\max)}$ *- greatest possible entropy of a system.*

The theorem 5 *Information capacity of technological system is defined by its stochastic part.*

The theorem 6 *The limiting degrees of determination and of ineradicable stochastic of technological system are defined under the formula:*

$$d_{\sum_n} = \frac{I_{\sum_n}(d)}{H_{\sum n(\max)}}, \quad h_{\sum_n} = \frac{I_{\sum_n}(h)}{H_{\sum_{n(\max)}}},$$

where $I_{\sum_n}(d), I_{\sum_n}(h)$ *- a system determined and stochastic components,* $H_{\sum_{n(\max)}}$ *- the system maximal information.*

For the limiting characteristics of technological system the degree of determination is equal to factor of the redundancy $R = 1 - \frac{H_r}{H_{\max}}$, used in the theory of the information; $H_{\max}$ - greatest possible entropy of a system, H_r - entropy of a system during the considered moment.

For limiting characteristics of technological system the relation $\frac{h}{d}$ is equal to factor of the stochasticity $G = \frac{H_r}{I_r} = \frac{1-R}{R}$, used in the theory of the information; I_r - the realized information, H_r - entropy of a system during the considered moment, R - factor of redundancy.

The sense of surplus information is connected to knowledge of technological system, at which the taken information is always less than the information objectively contained in it in the form of the determined ratio. The factor of stochasticity can change from zero to infinite and in more details reflects the stochastic and determined properties of technological system [2]. The size equal to $\frac{1}{G} = Q$, gives a more evident representation of opportunities of unpredictable development of technological system, therefore by analogy to factor of stochasticity it can be called factor of determinacy.

With the purpose of reception of analytical dependence of maximum stochastic information from the general conditions of formation of technological system from the formula (6) with a method of a mathematical induction we shall deduce the recurrent formula for a finding $I_n(h)$:

$$I_n(h) = \frac{H_{n(\max)}}{(n+1)!} = \frac{k_n}{n+1} I_{n-1}(h).$$

At substitution of equality (2) in (6) we shall receive formulas for definition of all kinds of the information of hierarchical system:

$$I_n(h)=\frac{H_{n(\max)}}{(n+1)!}=\frac{k^n \log N}{(n+1)!}, \tag{7}$$

$$I_{\sum_n}(h)=\sum_{i=0}^{n}\frac{H_{i(\max)}}{(i+1)!}=\log N\sum_{i=0}^{n}\frac{k^i}{(i+1)!}, \tag{8}$$

$$I_n(d)=H_{n(\max)}\left[1-\frac{1}{(n+1)!}\right]=k^n\left[1-\frac{1}{(n+1)!}\right]\log N, \tag{9}$$

$$I_{\sum_n}(d)=\sum_{i=0}^{n}H_{n(\max)}\left[1-\frac{1}{(n+1)!}\right]=\log N\sum_{i=0}^{n}k^i\left[1-\frac{1}{(i+1)!}\right], \tag{10}$$

$$H_{n(\max)}=k^n \log N, \tag{11}$$

$$H_{\sum n(\max)}=\log N\sum_{i=0}^{n}k^i . \tag{12}$$

From formulas for the determined component and maximal information of technological system follows, that they have no final limits at $n \to \infty$ and are unlimited functions. Concerning a stochastic part (8), we have a numerical line converging in indication of d´Alamber. For the proof of convergence we shall make a limit of the relation of a $n+1$member to n - member of a line:

$$\lim_{n\to\infty}\frac{k^{n+1}\cdot(n+1)!}{(n+2)!\cdot k^n}=\lim_{n\to\infty}\frac{k}{n+2}=0<1\cdot$$

It means, the stochastic component of the information at self-organizing hierarchical systems is limited [4].

From the proof of theorems 2, 3 follows, that the most thin changes of the general informational capacity of system are connected to a stochastic part. And though restrictions on absolute size of the determined information are absent, nevertheless $I_{\sum_n}(d)$ on size will never reach $H_{\sum n(\max)}$, differing from it on size $I_{\sum_n}(h)$. Therefore boundary conditions for h, d, R, G, Q will be $0<h\le 1,\quad 0\le d=R<1,\quad G>0,\quad Q\ge 0$.

Thus, the theorems proved in the given section show indissoluble connection of the determined and stochastic components from which the first is dominating and providing stability, and the second defines the most thin changes and optimum information capacity of technological systems. In this connection we conclude, that the entropy-information approach to research of technological systems is objectively necessary. In the technological circuit considered by us $k=2$ there is a sample of set of elements - an element and not an element (in our case copper and all other elements in aggregate) then the equation (2) will become:

$$H_{n(\max)}=2^n \log N_0=2^n \log_2 2=2^n .$$

On the basis of the theorem 1 we shall calculate the maximal information of the technological circuit on initial 10 levels at $k=2$:

n	0	1	2	3	4	5	6	7	8	9	10
$H_{n(\max)}$ bit/el.	1	2	4	8	16	32	64	128	256	512	1024

Essentially important advantage of an information estimation of quality of products or technological operations is that a suggested parameter H_n, as well as any entropy-information sizes, can be added. The given property of additive is immanently inherent to entropy and information and is a basis for expression of the law of preservation of their sum. Hence, technological uncertainty of various operations within the limits of the unified circuit can be expressed by a system parameter of uncertainty:

n	0	1	2	3	4	5	6	7	8	9	10
$H_{\sum n(\max)}$, bit/el.	1	3	7	15	31	63	127	255	511	1023	2047

$$H_{\sum n(\max)} = \sum_{i=0}^{n} H_i = \sum_{i=0}^{n} 2^i \text{, bit/el.}$$

The determined component of the information on the basis of the theorem 2 is defined by:

$$I_n(d) = 2^n \left[1 - \frac{1}{(n+1)!}\right] \text{ bit/el.}$$

n	0	1	2	3	4	5	6	7	8	9	10
$I_n(d)$, bit/el.	0	1	3,33	7,67	15,9	32,0	64,0	128	256	512	1024

As the information capacity of technological system is defined by its stochastic part on the basis (3) we shall receive:

$$I_n(h) = \frac{2^n}{(n+1)!} \text{ bit/el.,}$$

n	0	1	2	3	4	5	6	7	8	9	10
$I_n(h)$, bit/el.	1	1	0,6667	0,3333	0,1333	0,0444	0,0127	0,0032	0,0007	0,0001	0,0000

The system determined component $I_{\sum_n}(d)$ is equal:

$$I_{\sum_n}(d) = \sum_{i=0}^{n} 2^i \left[1 - \frac{1}{(i+1)!}\right] \text{ bit/el.,}$$

n	0	1	2	3	4	5	6	7	8	9	10
$I_{\sum_n}(d)$, bit/el.	0	1	4,33	12	27,9	59,8	124	252	508	1020	2044

Having defined degrees of determination and ineradicable stochasticity at each level of technological system under formulas [3]:

$$d_n = \frac{I_n(d)}{H_{n(\max)}}, \quad h_n = \frac{I_n(h)}{H_{n(\max)}} = 1 - d,$$

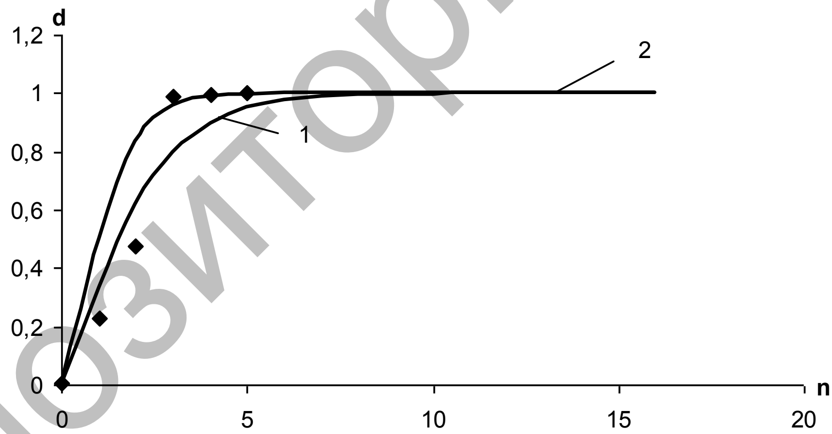
let's analyze the received results of the carried out calculations which are submitted in table 2.

We shall illustrate the comparison of these data with practical "know-how" of copper (tab. 1) graphically in coordinates n, d. The factor of their correlation (d_n) has made 0,8614 at the importance 6,6744>2, and ($d_{\sum_n}$) has made 0,9479 at the importance 7,348>2, that testifies the adequacy of suggested model of an information estimation of quality of products in consecutive operations of the technological circuit.

Table 2. Settlement information-entropy characteristics of technological repartitions in hierarchical system for $k = 2$, $N_0 = 2$

n	$I_n(d)$	$H_{n(\max)}$	$d_n = \frac{I_n(d)}{H_{n(\max)}}$	$I_{\sum_n}(d)$	$H_{\sum_{n(\max)}}$	$d_{\sum_n} = \frac{I_{\sum_n}(d)}{H_{\sum_{n(\max)}}}$
0	0	1,0	0	0	1,0	0
1	1,00	2,0	0,50	1,00	3,0	0,33
2	3,33	4,0	0,83	4,33	7,0	0,62
3	7,67	8,0	0,96	12,0	15,0	0,80
4	15,9	16,0	0,99	27,9	31,0	0,90
5	32,0	32,0	1,0	59,8	63,0	0,95
6	64,0	64,0	1,0	124,0	127,0	0,98
7	128,0	128,0	1,0	252,0	255,0	0,99
8	256,0	256,0	1,0	508,0	511,0	0,99
9	512,0	512,0	1,0	1020,0	1023,0	0,998
10	1024,0	1024,0	1,0	2044,0	2047,0	0,999

The size N in this case does not influence the solution of a problem as it is reduced at calculation of level d_n and system $d_{\sum_n}$ determinations.



1,2 - dependence on new model, points - experimental data

Fig. 1 Dependence of a degree of determination on a level

Influence of length of a code k that is elements of system (target component and the basic impurity) can be revealed in the further researches. As a whole the improvement of quality of a product in process of its technological processing correlates with dynamics of growth of the determined component in abstract hierarchical system that proves the expediency of the further entropy-information analysis of similar systems.

Conclusions

1 For the information analysis of quality of technological products and processes of their reception quantitative estimations of value of the information can be made only after the preliminary arrangement what exactly in each concrete case has value for considered metallurgical processes.

2 The strict mathematical substantiation of all calculations in the form of the formulated and proved 6 theorems in addition to earlier general formulas on the entropy-information analysis of self-organizing hierarchical systems published by professor Malyshev V.P. The suggested theorems can be of interest, as from the point of view of the theoretical analysis of various technological circuits, and concerning the further development of entropy-information representations and display of any objects.

3 Use of the measure of certainty and uncertainty of the information allows to analyze the general mechanisms of entropy-information laws of the technological repartitions being a fundamental basis of all spontaneously proceeding processes of accumulation of the information which result in self-organizing technological processes, namely, to hierarchical systems.

4 For multilevel hierarchical system of technological repartition it is important to describe the subordinate level as interaction of the interconnected subsystems, each of which possesses the information properties. Therefore at reception of an information estimation main attention is inverted on into-level and intra-level interactions. The considered approach, in our opinion, fully complies with the basic requirements of the system entropy-information analysis as while modeling hierarchical system of technological processes it provides integrity of its consideration due to the general-theoretical and methodical concepts allowing to keep in sight the system as a whole entirely for the solution of a task at all levels of hierarchical system.

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