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Тұтынушылардың ақпараттық-телекоммуникациялық білім технологиялары сапасымен қанағаттандырылуын бағалау

Мақалада тұтынушылардың қанағаттандырылуын анықтайтын бағалау әдісі мен сәйкес сауалнамадан құралған бағалау жүйесі келтірілген. Ақпараттық-телекоммуникациялық білім технологияларынының сапасын бағалау сауалнамалардың нәтижелерінің негізінде анықталған.

Б.М.Нурланова

Оценка удовлетворенности потребителей качеством информационно-телекоммуникационных образовательных технологий

В статье разработана система оценивания, содержащая метод оценки и соответствующую анкету для определения удовлетворенности потребителей. Оценка качества информационно-телекоммуника-ционных образовательных технологий определяется на основе обработки результатов анкетирования.

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Numerical solution of initial-boundary problem for the Helmholtz equation on «discretization – optimization»

In this paper we consider the solution of inverse problems on a «discretization — optimization». Considering the direct problem in discrete form, we calculate the functional gradient in a discrete form, using the formula for summation by parts, we obtain the formulation of the conjugate problem in discrete form. Construct an algorithm for solving the inverse problem. Numerically solve the inverse problem. And also performed numerical calculations for the solution of inverse problems.

Key words: numerical solution, initial-boundary problem, discretization – optimization, Helmholtz equation, inverse problem, Landweber iteration method.

Introduction

In this paper [1] considered the Cauchy problem for the Helmholtz equation, the authors present the theoretical research of the problem. Solution of the problem is considered scheme «optimization – discretization». Initial problem is reduced to the inverse problem, which is written in operator form. Operator equation reduces to the problem of minimizing the objective functional. We write the functional gradient.

Construct an algorithm for solving the inverse problem. For the numerical solution using discretization direct and conjugate problems.

Numerical solution of initial-boundary problem for the Helmholtz equation on «optimization – discretization»

Consider an algorithm to solve the inverse problem scheme «optimization – discretization».

Algorithm to solve the inverse problem using the method Landweber.

1. Choose an initial approximation q_0.
2. Suppose we know q_n, then numerically solve the direct problem:

$$u_{xx}+u_{yy}+k^2u=0,\quad (x,y)\in\Omega;$$
$$u_x(0,y)=0,\quad y\in[0,\pi];$$
$$u(l,y)=q(y),\quad y\in[0,\pi];$$
$$u_y(x,0)=u_y(x,\pi)=0,\quad x\in[0,l],$$

where k — datum constant.

3. Calculate the value of the functional: $J(q)=\|Aq-f\|^2=\int_0^\pi\left[u(0,y;q)-f(y)\right]^2dy$.

4. If the value of the objective functional is not sufficiently small, then numerically solve the conjugate problems:

$$\psi_{xx}+\psi_{yy}+k^2\psi=0,\quad (x,y)\in\Omega;$$
$$\psi(l,y)=0,\quad y\in[0,\pi];$$
$$\psi_x(0,y)=2(u(0,y)-f(y)),\quad y\in[0,\pi];$$
$$\psi_y(x,\pi)=\psi_y(x,0)=0,\quad x\in[0,l].$$

5. Calculate the gradient of the functional $J'q=\psi_x(l,y)$.
6. Calculate the next approximation $q_{n+1}=q_n-\alpha J'q_n$ and go to step 2.

Here $\alpha=\left(0,\dfrac{1}{\|A\|^2}\right)$ [2].

Discretization of the inverse problem

For the numerical solution of the direct problem, first consider the discrete formulation of the problem. We construct in Ω a grid ω_h with step $h_x=\dfrac{l}{N_x},h_y=\dfrac{\pi}{N_y}$, where N_x,N_y — positive integers. Then $\omega_h=\left\{x=ih_x,y=jh_y;i=\overline{0,N_x},j=\overline{0,N_y}\right\}$. Corresponding discrete form of the direct problem for Helmholtz equations

$$\frac{u_j^{i+1}-2u_j^i+u_j^{i-1}}{h_x^2}+\frac{u_{j+1}^i-2u_j^i+u_{j-1}^i}{h_y^2}+k^2u_j^i=0; \qquad (1)$$

$$u_j^1-u_j^0=0; \qquad (2)$$

$$u_j^{N_x}=q_j; \qquad (3)$$

$$u_1^i-u_0^i=u_{N_y}^i-u_{N_y-1}^i=0. \qquad (4)$$

Objective functional has the form

$$J(q)=\sum_{j=0}^{N_y-1}\left[u_j^0-f_j\right]^2\cdot h_y.$$

Corresponding discrete form of the conjugate problem:

$$\frac{1}{h^2}\left(\psi_j^{i+1}-2\psi_j^i+\psi_j^{i-1}\right)+\frac{1}{h^2}\left(\psi_{j+1}^i-2\psi_j^i+\psi_{j-1}^i\right)+k^2\psi_j^i=0; \quad (5)$$

$$\psi_j^1-\psi_j^0=2\left(u_j^0-f^j\right); \quad (6)$$

$$\psi_j^{N_x}=0; \quad (7)$$

$$\psi_1^i-\psi_0^i=\psi_{N_y}^i-\psi_{N_y-1}^i=0. \quad (8)$$

Gradient of the objective functional has the form

$$J'(q)=\frac{\psi_{N_x,j}-\psi_{N_x-1,j}}{h_y}.$$

Numerical solution of initial-boundary problem for the Helmholtz equation on «discretization — optimization»

We consider the problem (1)–(4):

$$\frac{u_j^{i+1}-2u_j^i+u_j^{i-1}}{h_x^2}+\frac{u_{j+1}^i-2u_j^i+u_{j-1}^i}{h_y^2}+k^2u_j^i=0;$$

$$u_j^1-u_j^0=0,$$

$$u_j^{N_x}=q_j,$$

$$u_1^i-u_0^i=u_{N_y}^i-u_{N_y-1}^i=0.$$

Objective functional we approximate in the form:

$$J(q)=\sum_{j=0}^{N_y-1}\left[u_j^0-f_j\right]^2\cdot h_y.$$

We specify increment $q_j+\delta q_j$, then

$$u_j^i\approx u(x,y;q_n);$$

$$\tilde{u}_j^i\approx u(x,y;q_j^n+\delta q_j^n);$$

$$\delta u_j^i=\tilde{u}_j^i-u_j^i. \quad (9)$$

Using the notation (9), we calculate increment of the functional $J(q)$.

$$J(q+\delta q)-J(q)=\sum_{j=0}^{N_y-1}[\tilde{u}_j^0-f_j]^2\cdot h_y-\sum_{j=0}^{N_y-1}[u_j^0-f_j]^2\cdot h_y=\sum_{j=0}^{N_y-1}\delta u_j^0\cdot 2[u_j^0-f_j]\cdot h_y+o\left(\|\delta u\|\right). \quad (10)$$

For expressions to δu_j^0 we consider the formulation of the perturbed problem for equations (1)–(4):

$$\frac{1}{h_x^2}\left(\tilde{u}_j^{i+1}-2\tilde{u}_j^i+\tilde{u}_j^{i-1}\right)+\frac{1}{h_y^2}\left(\tilde{u}_{j+1}^i-2\tilde{u}_j^i+\tilde{u}_{j-1}^i\right)+k^2\tilde{u}_j^i=0; \quad (11)$$

$$\tilde{u}_j^1-\tilde{u}_j^0=0; \quad (12)$$

$$\tilde{u}_j^{N_x}=q_j+\delta q_j; \quad (13)$$

$$\tilde{u}_1^i-\tilde{u}_0^i=\tilde{u}_{N_y}^i-\tilde{u}_{N_y-1}^i=0. \quad (14)$$

From (11)–(14) we subtract relations (1)–(4) and granting this (9), we obtain for the increment δu_j^i of the problem:

$$\frac{1}{h_x^2}\left(\delta u_j^{i+1}-2\delta u_j^i+\delta u_j^{i-1}\right)+\frac{1}{h_y^2}\left(\delta u_{j+1}^i-2\delta u_j^i+\delta u_{j-1}^i\right)+k^2\delta u_j^i=0; \quad (15)$$

$$\delta u_j^1-\delta u_j^0=0; \quad (16)$$

$$\delta u_j^{N_x} = \delta q_j; \tag{17}$$

$$\delta u_1^i - \delta u_0^i = \delta u_{N_y}^i - \delta u_{N_y-1}^i = 0. \tag{18}$$

Summation by parts formula: $\Delta^i v = v^{i+1} - v^i$

- $(\Delta^i v, w^i) = v^N w^{N-1} - v^1 w^0 - (v^i, \Delta^{i-1} w)$;
- $(\Delta^{i-1} v, w^i) = v^{N-1} w^N - v^0 w^1 - (v^i, \Delta^i w)$.

Multiplying (15) by an arbitrary function ψ_j^i, sum over i from 1 to $N_x - 1$ and j from 1 to $N_y - 1$.

$$0 = h_x \cdot h_y \cdot \sum_{i=1}^{N_x-1}\sum_{j=1}^{N_y-1}\left[\frac{1}{h_x^2}\left(\delta u_j^{i+1} - 2\delta u_j^i + \delta u_j^{i-1}\right) + \frac{1}{h_y^2}\left(\delta u_{j+1}^i - 2\delta u_j^i + \delta u_{j-1}^i\right) + k^2 \delta u_j^i\right]\psi_j^i =$$

$$= \frac{h_y}{h_x}\cdot\sum_{j=1}^{N_y-1}\left[(\Delta^i \delta u_j, \psi_j^i) - (\Delta^{i-1}\delta u_j, \psi_j^i)\right] + \frac{h_x}{h_y}\cdot\sum_{i=1}^{N_x-1}\left[(\Delta_j \delta u^i, \psi_j^i) - (\Delta_{j-1}\delta u^i, \psi_j^i)\right] + k^2 h_x h_y \sum_{j=1}^{N_y-1}\sum_{i=1}^{N_x-1}\delta u_j^i \psi_j^i =$$

$$= \frac{h_y}{h_x}\cdot\sum_{j=1}^{N_y-1}\left[\delta u_j^{N_x}\psi_j^{N_x-1} - \delta u_j^1 \psi_j^0 - (\delta u_j^i, \Delta^{i-1}\psi_j) - \delta u_j^{N_x-1}\psi_j^{N_x} + \delta u_j^0 \psi_j^1 + (\delta u_j^i, \Delta^i \psi_j)\right] +$$

$$+ \frac{h_x}{h_y}\cdot\sum_{i=1}^{N_x-1}\left[\delta u_{N_y}^i \psi_{N_y-1}^i - \delta u_1^i \psi_0^i - (\delta u_j^i, \Delta_{j-1}\psi^i) - \delta u_{N_y-1}^i \psi_{N_y}^i + \delta u_0^i \psi_1^i + (\delta u_j^i, \Delta_j \psi^i)\right] + k^2 h_x h_y \sum_{i=1}^{N_x-1}\sum_{j=1}^{N_y-1}\delta u_j^i \psi_j^i =$$

$$= \frac{h_y}{h_x}\cdot\sum_{j=1}^{N_y-1}\left[\delta u_j^{N_x}\psi_j^{N_x-1} - \delta u_j^{N_x-1}\psi_j^{N_x} + \delta u_j^0(\psi_j^1 - \psi_j^0) + (\delta u_j^i, \Delta^i \psi_j - \Delta^{i-1}\psi_j)\right] +$$

$$+ \frac{h_x}{h_y}\cdot\sum_{i=1}^{N_x-1}\left[\delta u_{N_y}^i(\psi_{N_y-1}^i - \psi_{N_y}^i) + \delta u_0^i(\psi_1^i - \psi_0^i) + (\delta u_j^i, \Delta_j \psi^i - \Delta_{j-1}\psi^i)\right] + k^2 h_x h_y \sum_{i=1}^{N_x-1}\sum_{j=1}^{N_y-1}\delta u_j^i \psi_j^i =$$

$$= h_x h_y \sum_{i=1}^{N_x-1}\sum_{j=1}^{N_y-1}\left[\frac{1}{h_x^2}(\psi_j^{i+1} - 2\psi_j^i + \psi_j^{i-1}) + \frac{1}{h_y^2}(\psi_{j+1}^i - 2\psi_j^i + \psi_{j-1}^i) + k^2\psi_j^i\right]\delta u_j^i +$$

$$+ h_y \cdot \sum_{j=1}^{N_y-1}\left[\frac{\delta u_j^{N_x}\psi_j^{N_x-1} - \delta u_j^{N_x-1}\psi_j^{N_x} + \delta u_j^0(\psi_j^1 - \psi_j^0)}{h_x}\right] + h_x \sum_{i=1}^{N_x-1}\left[\frac{\delta u_0^i(\psi_1^i - \psi_0^i) - \delta u_{N_y}^i(\psi_{N_y}^i - \psi_{N_y-1}^i)}{h_y}\right].$$

Whence it follows the conjugate problem formulation the discrete form:

$$\frac{1}{h^2}(\psi_j^{i+1} - 2\psi_j^i + \psi_j^{i-1}) + \frac{1}{h^2}(\psi_{j+1}^i - 2\psi_j^i + \psi_{j-1}^i) + k^2\psi_j^i = 0; \tag{19}$$

$$\psi_j^1 - \psi_j^0 = 2(u_j^0 - f^j); \tag{20}$$

$$\psi_j^{N_x} = 0; \tag{21}$$

$$\psi_1^i - \psi_0^i = \psi_{N_y}^i - \psi_{N_y-1}^i = 0. \tag{22}$$

Then, using (10), we obtain the gradient of the functional

$$J'(q) = \frac{\psi_{N_x,j} - \psi_{N_x-1,j}}{h_y}. \tag{23}$$

Here ψ_j^i is a solution of the conjugate problem (19)–(22).

As we see the conjugate problem formulation (19)–(22) coincided formulation the conjugate problem (5)–(8).

Description of the numerical experiment

Suppose $l = 1$, $N_x = N_y = 20$, we choose a parameter $k = 0.9$. For the experiment, we take a piecewise constant function:

$$q_T(y) = \begin{cases} 0.3, & 0 < y \le \dfrac{\pi}{4} \\ 0.7, & \dfrac{\pi}{4} < y \le \dfrac{\pi}{2} \\ 0.8, & \dfrac{\pi}{2} < y \le \dfrac{3\pi}{4} \\ 0.2, & \dfrac{3\pi}{4} < y \le \pi \end{cases}$$

Specify the initial approximation $\tilde{q}_0(y) = 0.1$, try to restore the original exact solution. Landweber iteration step we define $\alpha = 0.01$.

Computational experiment no noise ($\varepsilon = 0$)

We consider a computational experiment, when data of the problem are given exactly. In table (1) presents the results of the calculation with the time of execution.

Table 1

Results solutions Landweber iteration method no noise

Number iteration, n	$\|q_T - \tilde{q}\|$	$J(q)$	Runtime, c
10	0.3185	0.006	4
100	0.2327	$5.13 \cdot 10^{-6}$	40
200	0.2302	$3.27 \cdot 10^{-6}$	80
257	0.2294	$3.00 \cdot 10^{-6}$	108

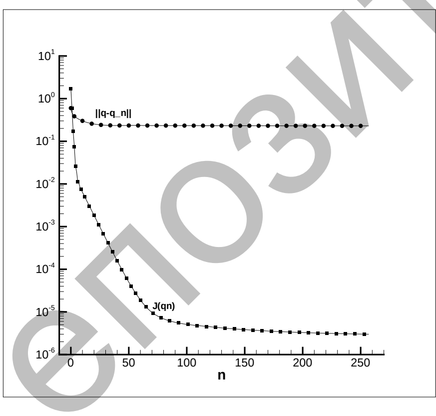

Figure 1. (Sign ▼) — norm difference exact function and reduced function $\|q_T - \tilde{q}\|$, (sign ▲) — value functional $J(q_n)$

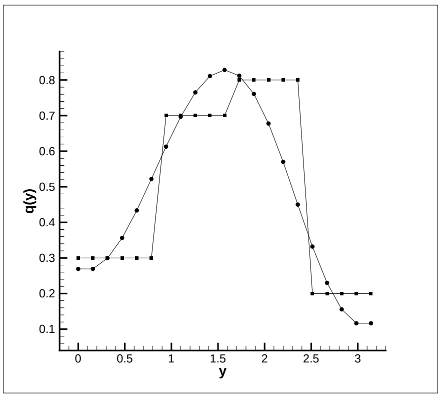

Figure 2. (Sign ▼) — exact solution $q(y)$, (sign ▲) — solution method Landweber

Figure 1 Comparison of the values of the functional and the error function. Figure 2 showing graphs of the exact and approximate solutions.

Computational experiment with noise ($\varepsilon = 0.01$)

Figures (3) and (4) and the table (2) shows the calculations, when $\varepsilon = 0.01$.

Table 2

Results solutions Landweber iteration method with noise 1%

Number iteration, *n*	$\|q_T - \tilde{q}\|$	$J(q)$	Runtime, c
0	0.5999	1.712	0.4
10	0.3028	$6.19 \cdot 10^{-3}$	4
20	0.2534	$1.88 \cdot 10^{-3}$	8
37	0.2339	$3.23 \cdot 10^{-4}$	15

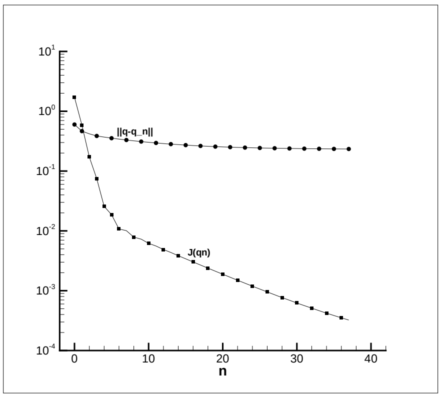

Figure 3. (Sign ▼) — norm difference exact function and reduced function $\|q_T - \tilde{q}\|$, (sign ▲) — value functional $J(q_n)$

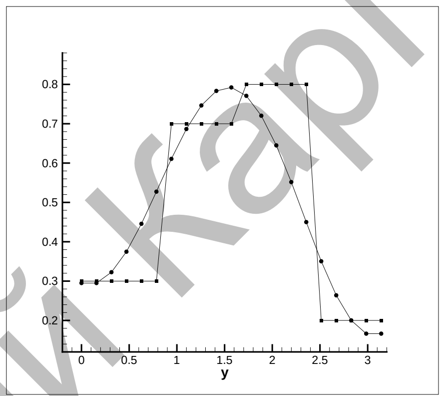

Figure 4. (Sign ▼) — exact solution $q(y)$, (sign ▲) — solution method Landweber

Figure 3 Comparison of the values of the functional and the error function. Figure 4 showing graphs of the exact and approximate solutions.

In the numerical solution of inverse problems using two schemes, the first scheme of «optimization — discretization» and the second scheme «discretization – optimization». In it was shown, that the scheme «discretization – optimization» the most effective, than the scheme «optimization – discretization». In our paper we show that, for the continuation problem for the Helmholtz equation, the two schemes for solving inverse problems identically, because formulation of the continuation problem (19)–(22) on a «discretization – optimization» same as with the formulation of the continuation problem (5)–(8) under the scheme «optimization – discretization».

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«Дискреттеу – оңтайландыру» үлгі бойынша Гельмгольц теңдеуі үшін бастапқы-шектік есебінің сандық шешімі

Мақалада «дискреттеу – оңтайландыру» үлгі бойынша кері есепті шешу қарастырылған. Тура есепті дискретті түрде зерттеп, функционал градиенті дискретті түрде есептелді, бөліктеп қосудың формуласын пайдаланып, түйіндес есептің қойылымы дискретті түрде алынды. Кері есепті шешудің алгоритмін тұрғызылды. Кері есеп сандық әдіспен шешілді. Сонымен қатар кері есеп шешімі бойынша сандық есептеулер келтірілді.

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Численное решение начально-краевой задачи для уравнения Гельмгольца по схеме «дискретизация – оптимизация»

В работе рассмотрено решение обратных задач по схеме «дискретизация – оптимизация». Прямая задача рассмотрена в дискретном виде, вычислен градиент функционала в дискретном виде с использовавшем формулы суммирования по частям, получена постановка сопряженной задачи в дискретном виде. Построен алгоритм решения обратной задачи, численно решена обратная задача, а также проведены численные расчеты по решению обратных задач.

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