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Жалпыланған жылу беру коэффициентін есептеудің рекурренттік арақатынасы

Мақалада автормен топырақта жылудың таратылу есебі қарастырыла отырып, бірөлшемді есептің математикалық моделі ұсынылды. Жер топырағы температурасының өлшенген мәні мен жер бетіндегі ауа температурасы берілді. Кері бастапқы-аймақтық есеп қарастырылып, қосалқы есеп шығарылды, топырақтың қоршаған ортаға жылу беру коэффициентін есептеу үшін итерациялық әдіс анықталды.

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Рекуррентное соотношение расчета обобщенного коэффициента теплообмена

Автором в результате изучения задачи распространения тепла в грунте предложена математическая модель одномерной задачи. Заданы измеренное значение температуры грунта земли и температура воздуха на поверхности земли. Рассмотрена обратная начально-краевая задача, построена сопряженная задача, выведен итерационный метод расчета коэффициента теплоотдачи почвы в окружающую среду.

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Priori estimates for the solution of direct and adjoint problems

In this work the reverse coefficient task is considered. The system of joint equalizations of transfer of heat, in an array «underground — under earth layer of atmospheres — the active layer of soil» in soil is described by nonlinear differential equation of the second order. A priori estimates for decisions direct and adjoint tasks for the case, when the generalized coefficient of heat emission is equal to the permanent size, are concluded.

Key words: the coefficient of heate mission, direct an dattended tasks, apriories timations, Koshi's in equality.

It is decided the task [1] is:

$$C\frac{\partial\theta}{\partial t}=\frac{\partial}{\partial z}\left(\lambda\frac{\partial\theta}{\partial z}\right),\ z\in(0,H),\quad t\in(0,t_{\max});\tag{1}$$

$$\theta\big|_{t=0}=\theta_0(z),\ \theta\big|_{z=0}=T_1;\tag{2}$$

$$\lambda\frac{\partial\theta}{\partial z}\bigg|_{z=H}=-N(t)\left(\theta\big|_{z=H}-T_0(t)\right).\tag{3}$$

It is required to define the meaning $N(t)$ that is the generalized coefficient of heat exchange. The axis z directs upwards, the beginning of coordinates is on the unchanging layer of temperature of soil. In the capacity of additional entered basis it is given the air temperature and measured value of temperature of soil on a surface. It is considered the particular case, when $T_0(t)=T_b(t)$. That is, having the measures $C,\ \lambda,\ \varphi(x),\ T_1,\ T_b(t)$ and $T_g(t)$ it is required to define $N(t)$ and $\theta(z,t)$. Here $T_g(t)$ is the measured value of temperature of soil on the earth surface.

The task is considered by an iterative method. Here n is an iterative parameter. In this case $N(t)$ is determined by the iterative measures $N(t,n)$, $n=0,1,...$

The beginning meaning $N(t,0)$ is given and the following meanings $N(t,n)$ is determined from the condition of monotony of functional [2–4]

$$J(N)=\int_0^{t_{\max}}(\theta(H,t)-T_g(t))^2\,dt. \quad (4)$$

The following variants of meanings of the generalized coefficient are possible:

1. $N(t)=N=const.$

2. $N(t)=N+\overline{N}\sin\frac{\pi t}{12}.$

3. $N(t)=N_0+\sum_{k=1}^{m}\left(N_{sk}\sin\frac{k\pi t}{12}+N_{ck}\cos\frac{k\pi t}{12}\right),$

where m is the limited whole positive number.

The example of a priori estimations for decision of task (1)–(3) is given for the case, when $N(t)=N=const.$

Here the task (1)–(3) has this kind

$$C\frac{\partial\theta}{\partial t}=\frac{\partial}{\partial z}\left(\lambda\frac{\partial\theta}{\partial z}\right); \quad (5)$$

$$\theta\big|_{t=0}=\theta_0(z),\ \theta\big|_{t=0}=T_1; \quad (6)$$

$$\lambda\frac{\partial\theta}{\partial z}\bigg|_{z=H}+N\theta\big|_{z=H}=NT_b(t). \quad (7)$$

We will increase (5) by $\theta(z,t)$ and will integrate on z from 0 till H. After the single integration into the parts on a variable z we have the equality

$$\frac{1}{2}\int_0^H C\frac{\partial\theta^2}{\partial t}dz=\lambda\frac{\partial\theta}{\partial z}\theta\big|_{z=H}-\lambda\frac{\partial\theta}{\partial z}\theta\big|_{z=0}-\int_0^H\lambda\left(\frac{\partial\theta}{\partial t}\right)^2dz.$$

We'll mark $\theta-T_1$ again in θ. Then the second integral in the right part of the sign of equality converts into zero. Taking into accounting of the condition (6)–(7) and suggesting, that $C=const$ we will integrate the will get equality on t from 0 till arbitrary t. Then

$$\frac{1}{2}\int_0^H\theta^2(z,t;n)dz+\int_0^t d\tau\int_0^H\lambda\left(\frac{\partial\theta}{\partial t}\right)^2dzd\tau+N(n)\int_0^t\theta^2(H,\tau;n)d\tau=$$

$$=N(n)\int_0^t\theta(H,\tau;n)T_в(\tau)d\tau+\frac{1}{2}C\int_0^H\theta_0^2(z)dz.$$

Using Koshi's in equality, we have the conclusion

$$\frac{1}{2}C\int_0^H\theta^2(z,t)dz+\int_0^t d\tau\int_0^H\lambda\left(\frac{\partial\theta}{\partial z}\right)^2dzd\tau+\frac{1}{2}N\int_0^t\theta^2(H,\tau)d\tau\le$$

$$\le\frac{1}{2}N\int_0^{t_{\max}}T_в^2(\tau)d\tau+\frac{1}{2}C\int_0^H\theta_0^2(z)dz.$$

There is

$$C_1=\max\left\{\frac{1}{2}\int_0^{t_{\max}}T_b^2(\tau)d\tau,\frac{1}{2}C\int_0^H\theta_0^2(z)dz\right\}.$$

Then

$$\frac{C}{2}\|\theta\|^2+\int_0^t\left\|\sqrt{\lambda}\frac{\partial\theta}{\partial z}\right\|^2d\tau+\frac{N}{2}\int_0^t\theta^2(H,t)d\tau\le C_1(1+N). \quad (8)$$

We take the estimation of attended tasks also for the case when $N(t)=N=const$. In this case the initial differential task is written in such a way.

$$C\frac{\partial\theta}{\partial t}=\frac{\partial}{\partial z}\left(\lambda\frac{\partial\theta}{\partial z}\right); \tag{9}$$

$$\theta\big|_{t=0}=\theta_0(z),\ \theta\big|_{t=0}=T_1; \tag{10}$$

$$\lambda\frac{\partial\theta}{\partial z}\bigg|_{z=H}+N\theta\big|_{z=H}=NT_b(t). \tag{11}$$

And attended differential tasks have the type

$$C\frac{\partial\psi}{\partial t}+\frac{\partial}{\partial z}\left(\lambda\frac{\partial\psi}{\partial z}\right)=0; \tag{12}$$

$$\psi\big|_{t=T_{\max}}=0,\ \psi\big|_{z=0}=0; \tag{13}$$

$$\left(\lambda\frac{\partial\psi}{\partial z}+N\psi\right)\bigg|_{z=H}=2\left(\theta-T_g(t)\right)\big|_{z=H}. \tag{14}$$

We multiply the equality(12) by $\psi(z,t)$ and integrate on z from 0 till H. After the single integration into the parts we have the equality:

$$\frac{C}{2}\int_0^H\frac{\partial}{\partial t}\psi^2(z,t)dz+\lambda\frac{\partial\psi}{\partial z}\big|_{z=H}-\int_0^H\lambda\left(\frac{\partial\psi}{\partial z}\right)dz=0.$$

Now we integrate on t from productive t to $t_{\max}$ and accounting the beginning-boundary conditions from (13) and (14) we have

$$\frac{C}{2}\int_0^H\psi^2(z,t)dz+\int_t^{t_{\max}}d\tau\int_0^H\lambda\left(\frac{\partial\psi}{\partial z}\right)^2dz+N\int_t^{t_{\max}}\psi^2(H,t)d\tau=$$

$$=2\int_t^{T_{\max}}\psi^2(H,\tau)\big(\theta(H,\tau)-T_g(\tau)\big)d\tau.$$

Using to the right part of Koshi's «ε-inequality» we have

$$\frac{C}{2}\int_0^H\psi^2(z,t)dz+\int_t^{t_{\max}}\left\|\sqrt{\lambda}\frac{\partial\psi}{\partial z}\right\|^2d\tau+(N-\varepsilon)\int_t^{t_{\max}}\psi^2(H,\tau)d\tau\le$$

$$\le\frac{1}{\varepsilon}\int_0^{T_{\max}}\theta^2(H,\tau)d\tau+\frac{1}{\varepsilon}\int_0^{T_{\max}}T_g^2(H,\tau)d\tau.$$

For example, $\varepsilon=\frac{N}{2}$, then we obtain

$$\frac{C}{2}\|\psi\|^2+\int_t^{t_{\max}}\left\|\sqrt{\lambda}\frac{\partial\psi}{\partial t}\right\|^2d\tau+\frac{N}{2}\int_t^{t_{\max}}\psi^2(H,t)d\tau\le$$

$$\le\frac{2}{N^2}\int_0^{t_{\max}}\theta^2(H,\tau)d\tau+\frac{2}{N}\int_0^{t_{\max}}T_g^2(\tau)d\tau.$$

We designate $\frac{1}{4}C_2=\max\left\{C_1,\int_0^{T_{\max}}T_g^2(\tau)d\tau\right\}$, then in result we have

$$\frac{C}{2}\|\psi\|^2+\int_t^{T_{\max}}\left\|\sqrt{\lambda}\frac{\partial\psi}{\partial z}\right\|^2d\tau+\frac{N}{2}\int_t^{T_{\max}}\psi^2(H,\tau)d\tau\le C_2\frac{1+N}{N^2}. \tag{15}$$

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Тура және қосалқы есептерді шешу үшін априорлық бағалар

Мақалада кері коэффициенттік есеп қарастырылды. Жылуды жерде тасымалдау үшін және «атмосфераның жер топырақтық қабаты — топырақтың қызметтік қабаты» атты массивінде бірлескен теңдеулер жүйесі екінші ретті сызықты емес дифференциалды теңдеу арқылы сипатталды. Жалпыланған жылу беру коэффициенті тұрақты шамаға тең жайдайда, тура және қосалқы есептердің априорлық бағалары анықталды.

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Априорные оценки для решений прямой и сопряженной задач

В статье рассмотрена обратная коэффициентная задача. Система совместных уравнений переноса тепла в массиве «припочвенно-приземный подслой атмосферы — деятельный слой почвы» в почве описана нелинейным дифференциальным уравнением второго порядка. Выведены априорные оценки прямой и сопряженной задач для случая, когда обобщенный коэффициент теплоотдачи равен постоянной величине.