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The attenuation coefficient and the velocity of thermal and elastic waves in orthorhombic syngony anisotropic media classes 222 and mm2

The relevance of research of wave propagation patterns in elastic medium with thermomechanical effect is related to the need to solve theoretical and applied problems of geophysics, seismology, mechanics of composite materials, etc. Bound equations of motion and heat conduction equation differ by complexity and an abundance of physical and mechanical parameters. In connection with this there is a rapidly developing branch of mechanics of deformable solids - thermoelasticity. Within this framework, based on the use of certain physical and mechanical properties of anisotropic medium, we study bound thermal and mechanical fields. In this paper, based on the method of matriciant, we identify the types of dependencies of velocities and attenuation coefficients of bound thermoelastic waves of frequency; high-quality graphics of velocities and damping coefficients of frequency are constructed under changing the parameters of the medium (thermo-mechanical parameters, temperature and thermal conductivity).

Key words: Anisotropic medium, thermoelasticity, Fourier heat equation, harmonic waves, dispersion, periodic structure, matriciant.

Introduction

The dynamical theory of thermoelasticity is the study of dynamical interaction between thermal and mechanical fields in solid bodies and is of much importance in various engineering fields such as earthquake engineering, soil dynamics, aeronautics, nuclear reactors, ets. It is well known that the classical theory of thermoelasticity [1, 2] rests upon the hypothesis of the Fourier law of heat conduction, in which the temperature distribution is governed by a parabolic-type partial differential equation. The theory predicts that a thermal signal is felt instantaneously everywhere in a body. This is unrealistic from the physical point of view, especially for short-time responses. To account for the effect of thermal relaxation, generalized thermoelasticity has been formulated on the basis of a modified Fourier law such that the temperature distribution is governed by a hyperbolic-type equation. Accordingly, heat transport in solids is regarded as a wave phenomenon rather than a diffusion phenomenon.

The wave propagation in anisotropic inhomogeneous medium is considered. A new method of matricant has been developed. The method of matricant allows to investigate wave processing in anisotropic medium with various physical and mechanical properties [3–5].

The structure of matricant for the equation motion elastic media equations, equations of thermomechanical medium has been established. Wave propagation in infinite and finite periodical inhomogeneous media are studied.

The application of matricants method for non-destructive testing and wave propagation in thermo elastic media is considered [6].

In the paper [7], waves propagating along an arbitrary direction in a heat conducting orthotropic thermoelastic plate are presented by utilizing the normal mode expansion method in generalized theory of thermoelasticity with one thermal relaxation time. In the paper [8], authors studied the interaction of free harmonic waves with multilayered media in generalized thermoelasticity by utilizing the combination of the linear transformation formation and transfer matrix method approach. Solutions obtained are generaland pertain to several special cases. Of these mention: (a) dispersion characteristics for a multilayered.

A Matriciant Method

At the present days solving wide range theoretical and applied problems of continuum dynamics requires more thorough consideration of anisotropy and physical and mechanical properties. The main peculiarity of analyzing wave processes in anisotropic medium is inapplicability of physical interpretations and mathematical methods developed for isotropic medium. It is related to the fact that it is impossible to separate wave field to forward and back waves. The other essential difficulty is an existence of a lot of physical parameters.

The method of study is an analytical method based on developing matrix techniques to study dynamics of the elastic layered medium.

The main idea is to deduce initial equations of the continuous medium and equations describing wave propagation in medium, based on the method of separation of variables, (solutions are represented as plane waves) to the equivalent set of ordinary differential equations with variable coefficients and then build the structure of a Matriciant (normalized matrix of fundamental solutions).

The problems of wave propagation in anisotropic medium, propagation of electromagnetic, electro-elastic, piezoelastic waves in anisotropic dielectrics, propagation of waves in anisotropic elastic and thermo-elastic medium, propagation of waves in anisotropic dielectric medium with magneto-electric effects, and orthotropic planes are analyzed by using a matriciant method.

The main advantage of a Matriciant method is equality of describing wave processes under the presence of one or several physical effects: elastic, thermo-elastic, magneto-elastic, piezo-elastic and magneto-electric, piezo-magnetic and magneto-electric effects.

In S.K. Tleukenov's international publications, the structure of the equations of motion matriciants in inhomogeneous medium were defined [6–8]. These publications were the beginning of a completely new level of studying the dynamics of inhomogeneous medium with application of that method and corporate studying of waves different by nature in inhomogeneous and periodically inhomogeneous anisotropic medium.

Consequently, development of the studying techniques and constituting interpretations about wave behavior in anisotropic medium should be considered as one of the high priority problem in theoretical physics and mechanics of deformable solids.

The matrix formulation of the propagation of thermoelastic waves.

Propagation of thermoelastic waves in anisotropic media described by the equations of motion to be solved together with the Fourier heat equation and the equation of heat flow, which have the form:

$$\sigma_{ij,j} = \rho \ddot{U}_i, \tag{1}$$

$$\lambda_{ij} \frac{\partial \theta}{\partial x_j} = -q_i, \tag{2}$$

$$\frac{\partial q_i}{\partial x_i} = -i\omega\beta_{ij}\varepsilon_{ij} - i\omega\frac{c_\varepsilon}{T_0}\theta, \tag{3}$$

where σ_{ij} — stress tensor, ρ — density of the medium; λ_{ij} — thermal conductivity tensor; q_i — the vector of heat; ω — the angular frequency; β_{ij} — thermomechanical constants, $\beta_{ij} = \beta_{ji}$; ε_{ij} — the strain tensor, c_ε — specific heat at constant strain; $\theta = T - T_0$ — temperature increase compared with the temperature of the natural state T_0, $\left|\frac{\theta}{T_0}\right| \prec\prec 1$ for small deformations.

Physical and mechanical quantities are related by relation of Duhamel-Neumann:

$$\sigma_{ij} = c_{ijkl}\varepsilon_{kl} - \beta_{ij}\theta. \tag{4}$$

Here c_{ij} — the elastic parameters; $c_{ijkl} = c_{jikl} = c_{ijlk} = c_{klij}$; ε_{kl} — the tensor Cauchy for small deformations.

Equations (1)–(4) determine the relationship of mechanical stress and temperature as a function of the independent variables — the thermal field and deformation.

Thus, the relation (1)–(4) constitute a closed system of thermoelasticity equations, which describes the propagation of thermoelastic waves.

Based on the method of separation of variables in the case of a harmonic function of time:

$$\left[U_i(x,y,z,t);\;\; \sigma_{ij}(x,y,z,t);\, \theta;\, q_z\right] = \left[U_i(z), \sigma_{ij}(z),\, \theta;\, q_z\right] e^{i(\omega t - mx - ny)}. \tag{5}$$

The system of equations (1)–(4) reduces to a system of differential equations of first order with variable coefficients which describes the propagation of harmonic waves:

$$\frac{d\vec{W}}{dz} = B\vec{W}, \tag{6}$$

here $B = B\left[c_{ijkl}(z), \beta_{ij}(z), \omega, m, n\right]$ — coefficient matrix whose elements contain the parameters of the medium in which waves propagate thermo elastic; m, n-components of the wave vector $\tilde{\kappa}$.

The vector $\vec{W}$ has the form:

$$\vec{W}(x,y,z,t) = \left[u_z(z), \sigma_{zz}, u_x(z), \sigma_{xz}, u_y(z), \sigma_{yz}, \theta, q_z\right]^t \exp(i\omega t - imx - iny). \tag{7}$$

The symbol t indicates the transpose of the vector — a vector of strings — Column.

The heterogeneity of the medium is assumed along Z. In constructing the coefficient matrix B is used as a representation of the solution (5), the system of equations (1)–(4) are in the derivatives along the coordinate Z and the excluded components of the stress tensor is not included in the boundary conditions. The multiplier $\exp(i\omega t - imx - iny)$ is omitted throughout.

Solution of the problem

In the case of one dimensional thermoelastic wave propagation in orthorhombic syngony anisotropic medium coefficients matrix B (if medium parameters are constant) has the following form:

$$B_0 = \begin{pmatrix} 0 & b_{12} & b_{17} & 0 \\ b_{21} & 0 & 0 & 0 \\ 0 & 0 & 0 & b_{78} \\ 0 & -i\omega b_{17} & b_{87} & 0 \end{pmatrix}, \tag{8}$$

here, coefficients b_{ij} are given by:

$$b_{12} = \frac{1}{c_{33}}; \quad b_{17} = \frac{\beta_{33}}{c_{33}}; \quad b_{21} = -\omega^2\rho; \quad b_{87} = -i\omega\left(\frac{\beta_{33}^2}{c_{33}} + c_\varepsilon\right); \quad b_{78} = -\frac{1}{\lambda_{33}}.$$

Considering condition [5]:

$$\det\left|B - \lambda E\right| = 0, \tag{9}$$

for this problem we obtain characteristic equation of the following form:

$$\lambda^4 - B\lambda^2 + C = 0, \tag{10}$$

where $B = b_{12}b_{21} + b_{78}b_{87}$, $C = b_{21}b_{78}(i\omega b_{17}^2 + b_{12}b_{87})$

from (10) we obtain:

$$\lambda_{1,2}^2 = \frac{1}{2}(b_{12}b_{21} + b_{78}b_{87}) \pm \frac{1}{2}\sqrt{(b_{12}b_{21} - b_{78}b_{87})^2 - 4i\omega b_{17}^2 b_{21} b_{78}}. \tag{11}$$

If we concede that longitudinal elastic and heat waves propagate unbound that is thermomechanical parameters $\beta_{ij} = 0$, then roots of characteristic equation (3) will be equal to:

$$\lambda_{1,2} = \pm i\omega\sqrt{\frac{\rho}{c_{33}}}; \ \lambda_{3,4} = \pm\sqrt{\frac{i\omega\, c_\varepsilon}{\lambda_{33}}}. \tag{12}$$

The first root of the relation (12) gives velocity of longitudinal wave that propagates with attenuation; second relation determines heat wave.

From the relation (11) we get four roots of characteristic equation (10) having following form:

$$k_{1,2} = \pm\sqrt{\frac{a}{2}\left(1 + \frac{2b - \frac{c}{a}}{\sqrt{2}\sqrt{D - x}}\right) + \frac{1}{2}i\left(b - \frac{1}{\sqrt{2}}\sqrt{D - x}\right)}; \quad k_{3,4} = \pm\sqrt{\frac{a}{2}\left(1 - \frac{2b - \frac{c}{a}}{\sqrt{2}\sqrt{D - x}}\right) + \frac{1}{2}i\left(b + \frac{1}{\sqrt{2}}\sqrt{D - x}\right)}. \tag{13}$$

where $a = b_{12}b_{21}$; $b = b_{78}b_{87}$; $c = 4i\omega b_{17}^2 b_{21} b_{78}$; $D = \sqrt{\left(a^2 + b^2\right)^2 + \left(2ab - c\right)^2}$.

These roots have already taken into account an effect that elastic and heat waves are bound that is $\beta_{ij} \neq 0$.

Let's rewrite $k_{1,2}$ in (13) in the following form:

$$k_{1,2} = \pm\sqrt{x_1 + iy_1} = \sqrt[4]{x_1^2 + y_1^2}\,(Cos\psi + iSin\psi); \tag{14}$$

$$k_{1,2}=\pm\sqrt{x_1+iy_1}=\pm\frac{1}{\sqrt{2}}\left(\frac{y_1}{\sqrt{D_1+x_1}}+i\sqrt{D_1+x_1}\right), \tag{15}$$

where $D_1=\sqrt{x_1^2+y_1^2}$, $x_1=\frac{a}{2}\left(1+\frac{2b-\frac{c}{a}}{\sqrt{2}\sqrt{D-x}}\right)$; $y_1=\frac{1}{2}\left(b-\frac{1}{\sqrt{2}}\sqrt{D-x}\right)$.

Roots $k_{3,4}$ in (13) are equal:

$$k_{3,4}=\pm\sqrt{x_2+iy_2}=-\frac{1}{\sqrt{2}}\left(\frac{y_2}{\sqrt{D_2+x_2}}\pm i\sqrt{D_2+x_2}\right), \tag{16}$$

where $x_2=\frac{a}{2}\left(1-\frac{2b-\frac{c}{a}}{\sqrt{2}\sqrt{D-x}}\right)$; $y_2=\frac{1}{2}\left(b+\frac{1}{\sqrt{2}}\sqrt{D-x}\right)$.

In an explicit form roots (15) and (16) have following form:

$$k_{1,2}=\pm\sqrt{\frac{c_\varepsilon\omega}{2\lambda_{33}}}\left(1+i\right)\left[1+\frac{\lambda_{33}}{2}\left(\frac{i\omega\, c_\varepsilon\rho\lambda_{33}T_0^2+c_{33}^3}{\rho^2\,\omega^3\,\lambda_{33}^2\,T_0^2+\omega\, c_\varepsilon^2\, c_{33}^2}\right)\beta_{33}^2\right]; \tag{17}$$

$$k_{3,4}=\pm i\sqrt{\frac{\rho\omega^2}{c_{33}}}\left(1-\frac{i\omega}{2c_{33}\lambda_{33}}\left(\frac{\rho\omega c_{33}\,\lambda_{33}^2\,T_0^2-i\,c_\varepsilon\, c_{33}^2\,\lambda_{33}\,T_0}{\rho^2\,\omega^3\,\lambda_{33}^2\,T_0^2+\omega\, c_\varepsilon^2\, c_{33}^2}\right)\beta_{33}^2\right). \tag{18}$$

Let's consider root k_1 in relations (17).
Real and imaginary parts of this root are equal:

$$\mathrm{Re}\,k_1=\sqrt{\frac{c_\varepsilon\omega}{2\lambda_{33}}}\left[1+\frac{\lambda_{33}}{2}\left(\frac{c_{33}^3-\omega c_\varepsilon\rho\lambda_{33}T_0^2}{\rho^2\,\omega^3\,\lambda_{33}^2\,T_0^2+\omega\, c_\varepsilon^2\, c_{33}^2}\right)\beta_{33}^2\right]; \tag{19}$$

$$\mathrm{Im}\,k_1=i\sqrt{\frac{c_\varepsilon\omega}{2\lambda_{33}}}\left[1+\frac{\lambda_{33}}{2}\left(\frac{\omega c_\varepsilon\rho\lambda_{33}T_0^2+c_{33}^3}{\rho^2\,\omega^3\,\lambda_{33}^2\,T_0^2+\omega\, c_\varepsilon^2\, c_{33}^2}\right)\beta_{33}^2\right]. \tag{20}$$

From the imaginary part of the root k_1 we obtain formula for velocity of heat wave:

$$c=\frac{k_1}{\omega}=\sqrt{\frac{2\lambda_{33}\,T\omega}{c_\varepsilon}}\left[1-\frac{\lambda_{33}}{2}\left(\frac{\omega c_\varepsilon\rho\lambda_{33}T_0^2+c_{33}^3}{\rho^2\,\omega^2\,\lambda_{33}^2\,T_0^2+c_\varepsilon^2\, c_{33}^2}\right)\beta_{33}^2\right]. \tag{21}$$

Real part of this root allows to get attenuation coefficient of heat wave:

$$k_{зат}=\sqrt{\frac{c_\varepsilon\omega}{2\lambda_{33}}}\left[1+\frac{\lambda_{33}}{2}\left(\frac{c_{33}^3-\omega c_\varepsilon\rho\lambda_{33}T_0^2}{\rho^2\,\omega^2\,\lambda_{33}^2\,T_0^2+c_\varepsilon^2\, c_{33}^2}\right)\beta_{33}^2\right]. \tag{22}$$

Now, let's consider positive root k_3 in relation (18).

Real and imaginary parts of this root allows to get attenuation coefficient and velocity of elastic wave:

$$k_{зат}=\frac{1}{2}\sqrt{\frac{\rho}{c_{33}}}\left(\frac{\rho\,\omega^3\,c_{33}\,\lambda_{33}\,T_0^2}{\rho^2\,\omega^2\,\lambda_{33}^2\,T_0^2+c_\varepsilon^2\,c_{33}^2}\right)\beta_{33}^2; \tag{23}$$

$$c=\sqrt{\frac{c_{33}}{\rho}}\left(1-\frac{1}{2}\left(\frac{c_\varepsilon\,c_{33}\,T_0}{\rho^2\,\omega^2\,\lambda_{33}^2\,T_0^2+c_\varepsilon^2\,c_{33}^2}\right)\beta_{33}^2\right). \tag{24}$$

As a result of the roots (17), (18) high quality graphics, presented below, of dependencies of velocity and attenuation coefficients of elastic and heat wave from frequency are constructed under, changing the parameters of the medium (thermo-mechanical parameters, temperature).

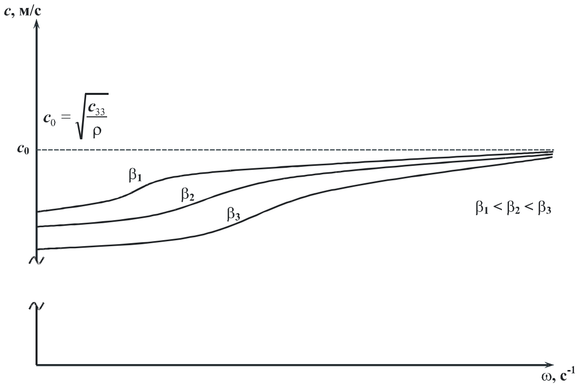


Figure 1. Diagram of velocity c of elastic longitudinal wave and frequency under different thermomechanical parameters β_{ij}

From the given diagram it can be seen that under increase of thermomechanical parameter velocity of longitudinal elastic wave decreases.

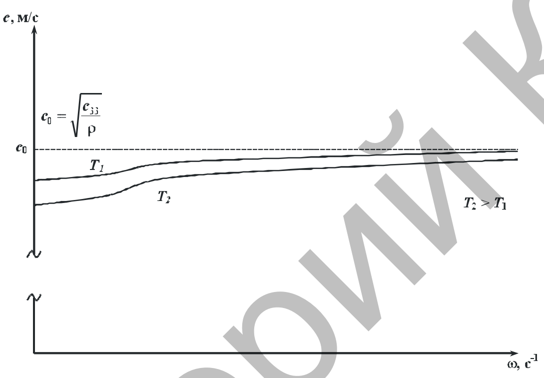


Figure 2. Elastic longitudinal wave velocity c and frequency diagram under different temperatures

This diagram indicates that an increase of thermodynamic temperature causes a decrease of velocity of elastic longitudinal wave. It's related to lattice site oscillation that affects wave velocity.

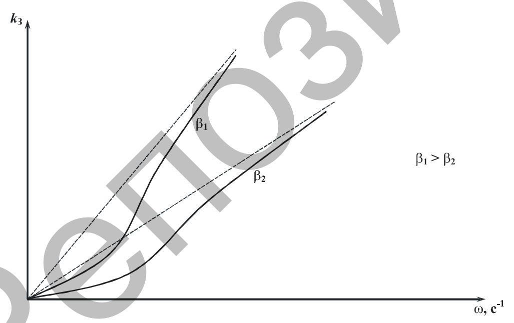
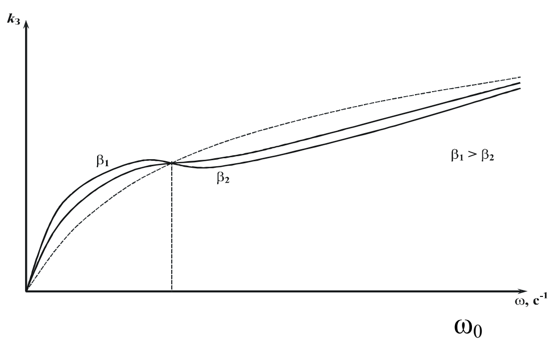


Figure 3. diagram of attenuation coefficient k_3 of elastic longitudinal wave and frequency under different thermomechanical parameters β_{ij}



Picture 4. diagram of attenuation coefficient k_3 of heat wave and frequency under different thermomechanical parameters β_{ij}

It follows from the last diagram that an increase of thermomechanical parameter causes attenuation of heat wave in anisotropic medium. Under explicit magnitude of frequency ω_0 which can be derived from equation (22) there is no interaction of heat and elastic waves that is these waves propagate without thermoelastic effect and this frequency will be valid under any thermomechanical parameter β_{ij}.

Conclusion

In this paper, based on the method of matriciant, we identify the types of dependencies of velocities and attenuation coefficients of bound thermoelastic waves of frequency; high-quality graphics of velocities and damping coefficients of frequency are constructed under changing the parameters of the medium (thermo-mechanical parameters, temperature and thermal conductivity).

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222 және mm2 класты ромбылық сингониялы анизотропты орталарда жылулық және серпімді толқындардың өшу және жылдамдық коэффициенттері

Термомеханикалық эффектімен болатын серпімді орталарда толқындық процестердің заңдылықтарды зерттеу өзектілігі, геофизика, сейсмология, композиттік материалдардың механикасының теориялық және қолданбалы есептерді шешуінде қажеттілігімен байланысты. Байланысқан қозғалыс теңдеулері мен жылуөткізгіштік теңдеулері физика-механикалық параметрлердің күрделігі мен көп болуымен ерекшеленеді. Осыған байланысты деформацияланатын қатты дене механикасының «Термосерпімділік» деген тарауы қарқынды дамып келеді. Осы бағыттың аясында анизотропты орталардың кейбір физика-механикалық қасиеттерін қолдана отырып, байланысқан жылулық және механикалық өрістер зерттелді. Мақалада, матрицант әдісінің негізінде, жиілікке тәуелді байланысқан термосерпімді толқындардың жылдамдықтары мен өшу коэффициенттерінің тәуелділіктердің түрлері анықталды; серпімді және жылу толқындардың (термомеханикалық параметрлердің аздығы кезіндегі) жылдамдықтардың және өшу коэффициентерінің температураның, жылуөткізiштік коэффициентінің және жиіліктің өзгерісіне тәуелділігінің сапалы графиктері сызылды.

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Коэффициенты затухания и скорости тепловых и упругих волн в анизотропной среде ромбической сингонии классов 222 и mm2

Актуальность исследования закономерностей волновых процессов в упругих средах с термомеханическим эффектом связана с необходимостью решения теоретических и прикладных задач геофизики, сейсмологии, механики композитных материалов и т.д. Связанные уравнения движения и уравнения теплопроводности отличаются сложностью и обилием физико-механических параметров. В связи с этим интенсивно развивается раздел механики деформируемого твердого тела «Термоупругость». В рамках этого направления, опираясь на использование определенных физико-механических свойств в анизотропных средах, изучаются связанные тепловые и механические поля. В статье, на основе метода матриканта, определены виды зависимостей скоростей и коэффициентов затухания связанных термоупругих волн от частоты; построены качественные графические зависимости скоростей и коэффициентов затухания упругих и тепловых волн от частоты при изменении параметров среды (термомеханического параметра, температуры и коэффициента теплопроводности).

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Сложный изгиб упругой неоднородной пластины в неравномерном температурном поле

В статье рассмотрена симметричная деформация неравномерно нагретой пластины, когда модуль упругости является переменным не только вдоль радиуса, но также и по толщине пластины. Развитие современной практики требует от исследователей и конструкторов создания новых методов решения большого числа прочностных задач, связанных с переменностью толщины, модуля упругости, коэффициента Пуассона, наличием высокого температурного поля в агрегатах и узлах конструкции. Высока потребность в аналитических и приближенно аналитических методах решения задач о расчетах напряженно-деформированного состояния неоднородных пластин в неравномерном температурном поле. Исследование таких задач исключительно актуально. В данной работе в существенной мере выполняется указанный пробел.

Ключевые слова: сложный изгиб, неоднородная пластина, упругость, неравномерное температурное поле, аналитическое решение.

Задача об изгибе упругих неоднородных пластин при переменных параметрах с учетом неравномерного температурного поля является одной из актуальных задач технической теории упругости. Анализ температурных напряжений и деформаций в конструктивных элементах различного типа двигателей и установок, работающих при высоких температурах, имеет исключительно большое значение. Точный расчет тонких пластин в условиях неравномерного нагрева с учетом изменения механических параметров материала весьма сложен, так как связан с решением систем нелинейных дифференциальных уравнений с переменными коэффициентами. Поэтому построение аналитического решения названной задачи является весьма актуальной.

Круглая пластина переменной толщины в качестве конструктивного элемента находит широкое применение, к расчету которой приводят многие вопросы, связанные с проектированием круглых фундаментных плит, турбинных дисков, гибких соединений валов и др.

В последние годы большое значение приобрел анализ температурных напряжений в различных конструкциях ядерных реакторов. Большую роль играет анализ температурных напряжений в тепловыделяющих элементах, что особенно важно для реакторов, работающих в условиях высоких темпе-