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The solution of a wave equation in two-dimension anti-de Sitter space

A solution of a wave equation on a surface of a one-sheet hyperboloid is found in the article. The surface is considered as two-dimension model of the accelerated Universe. It is shown the phase velocity increases in the global reference frame, but it is constant with regard to a concomitant one. Allowable frequencies and wave numbers are discrete and decreasing with time proportionally to the space curvature. The performed calculations show the discreteness step has to be much less than present-day experimental possibilities of measurements.

Key words: wave equation, anti-de Sitter space, metric tensor, expansion of the Universe.

Introduction

Recent achievements in the experimental cosmology lead to radical conclusions about necessity of serious revision of our theoretical knowledge about the Universe physics. The first indication to it was a discovery of a discrepancy between the Newton laws or General Relativity and visible mass distributions for spiral galaxies [1]. The solution of the problem supposed introduction of a new notion that called as dark matter [2]. It is hypothetical non radiating matter that also makes a deposit in the gravitational potential. Estimations for the dark matter quantity lead to a conclusion about the space geometry of the Universe as having very small curvature and going close to Euclidian space. However estimation for the rate of the Universe expansion based on analysis of super novas emittance leads to the conclusion about accelerated character of the process that cannot be understood if to take account only gravitational attractive forces [3]. There are some rather different approaches for explanation of the effect that was named as dark energy. The main is the Λ-item returning into Einstein equation. So repulsion forces appear that are increasing with distance. For explanation of dark matter there are proposed different kinds of exotic matter [4,5]. Also some alternative gravitational theories as Modified Newtonian Dynamics are actively elaborated by some researchers [6].

To the present day the question about geometry of the Universe is open. Observational flatness of the visible part of the Universe stipulates models which allow combine this property with a closed space constituent of the space-time (see [7] for example). The idea of the closed space doesn't have any serious physical foundation excluding difficulties with perception of the infinity Universe and some paradoxes which have quite acceptable explanations.

1. The model for the closed accelerated Universe

We will consider the Universe as a maximum symmetric subspace with less number of dimensions than the enveloping Minkovski flat space. For simplicity let us consider a toy-model with two space dimensions and one time one with flat metrics

$$ds^2 = dT^2 - dX^2 - dY^2 , \tag{1}$$

where T is a world time of the enveloping space-time (we put $c = 1$), X and Y are space Cartesian coordinates there. A two dimensional space modeling the Universe is the infinity cylinder oriented along the time. The Universe evolution implies that its radius R is a function of time.

Let us introduce an angle coordinate α in the next way

$$X = R\sin\alpha , \ Y = R\cos\alpha .$$

The differentials of the expressions have the next view

$$dX = dR\cos\alpha - R\sin\alpha\, d\alpha , \ dY = dR\sin\alpha - R\cos\alpha\, d\alpha .$$

Taking account that $dX^2 + dY^2 = dR^2 + R^2 d\alpha^2$ and $dR = \dfrac{\partial R}{\partial T} dT$ we have instead (1) the interval in the polar coordinate system

$$ds^2 = \left(1 - \left(\frac{\partial R}{\partial T}\right)^2\right) dT^2 - R^2 d\alpha^2 .$$

Note that the first item in the right-hand side is the squared proper time (when $\alpha = const$) on the cylinder surface which will be denoted as t. That is

$$dt = \sqrt{1 - \left(\frac{\partial R}{\partial T}\right)^2} dT . \quad (2)$$

Really the speed of a point in the enveloping space with $\alpha = const$ equals to $\partial R / \partial T$. Therefore the formula (2) correlates with a common expression of Special relativity for time dilation

$$\tau = \frac{\tau_0}{\sqrt{1 - v^2 / c^2}} ,$$

where τ_0 is a proper time. So the metric tensor on the cylinder has the next simple view

$$ds^2 = dt^2 - R^2 d\alpha^2 .$$

The next important moment has to be emphasized. In contrast to the time t that is measured directly on the surface, the space coordinate α demands the enveloping space consideration for its interpretation. Instead of this we can introduce the distance $x = \alpha R$ that is measured along the surface too and can be interpreted as the usual Cartesian coordinate but in the «pseudo-Cartesian reference frame» introduced on the cylinder. The coordinate α has to be interpreted as an «associated coordinate» like ones introduced in Cosmology.

From the last expressions the co- and contra-variant metric tensors can be written

$$g_{ij} = \begin{pmatrix} 1 & 0 \\ 0 & -R^2 \end{pmatrix}, \; g^{ij} = \begin{pmatrix} 1 & 0 \\ 0 & -1/R^2 \end{pmatrix}. \quad (3)$$

For solving of concrete problems the function $R(t)$ has to be chosen. Let us chose the cylinder in the view of some one-sheet hyperboloid (see Fig.) oriented along the time and put

$$R(T) = \sqrt{R_0^2 + T^2} . \quad (4)$$

Such choice is attractive by that fact that the surface is built by isotropic (in the enveloping space) straight lines [8] and the radius of the subspace accelerates with time with accordance to the acceleration of the Universe.

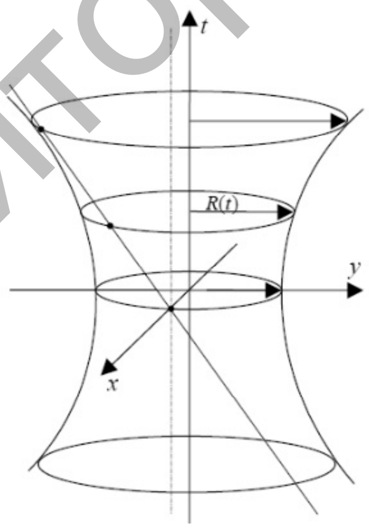

Figure. Schematic representation of the closed space-time used in the work

2. The wave equation in (1,1) anti-de Sitter space

In the general case of a non-Euclidian metric space with the connection is consisted with the metrics the Laplace equation has the view

$$\frac{1}{\sqrt{|g|}} \frac{\partial}{\partial x^i} \left(g^{ik} \sqrt{|g|} \frac{\partial f}{\partial x^k} \right) = 0 .$$

This expression can be considered as the wave equation if one of coordinates is dedicated as time one and the metric signature is appropriate. In our case the metrics has the view (3) and therefore $\sqrt{|g|}=R$. So,

$$R\frac{\partial}{\partial t}\left(R\frac{\partial f}{\partial t}\right)-\frac{\partial^2 f}{\partial\alpha^2}=0\,. \tag{5}$$

A solution of the equation will be finding in the view

$$f(\alpha,t)=g(\alpha)h(t)\,.$$

After introduction of the constant m^2 for separation of variables we get to the system

$$R\frac{\partial}{\partial t}\left(R\frac{\partial h}{\partial t}\right)=-m^2 h\,,$$

$$\frac{\partial^2 g}{\partial\alpha^2}=-m^2 g\,. \tag{6}$$

A solution of the last equation has the standard view

$$g(\alpha)=const\cdot\exp(im\alpha)\,.$$

The condition of unambiguity of the solution ($\alpha=\alpha+2\pi$) leads to the demand for the parameter m to be an integer number $m=0,\pm1,\pm2,\ldots$.

For solving of the time equation of the system (6) the function $R(t)$ has to be substituted. So, from (2) and (4) we have

$$dt=\sqrt{1-\frac{T^2}{R_0^2+T^2}}dT=\frac{R_0}{\sqrt{R_0^2+T^2}}dT\,.$$

Therefore for the time on the hyperboloid surface we have:

$$t=\int_0^T\frac{R_0}{\sqrt{R_0^2+T^2}}dT\,.$$

The origin of t is chosen to be coincided with the one on the enveloping space at $T=0$. So,

$$t=R_0\,\mathrm{arcsh}\frac{T}{R_0}\,,\quad T=R_0\,\mathrm{sh}\frac{t}{R_0}\,.$$

Therefore for the function $R(t)$ we have

$$R(t)=R_0\,\mathrm{ch}\frac{t}{R_0}\,.$$

After substitution it into the first equation of the system (6) we get to the equation

$$\mathrm{ch}\,\tau\frac{\partial}{\partial\tau}\left(\mathrm{ch}\,\tau\frac{\partial h}{\partial\tau}\right)+m^2h=0\,,$$

where the dimensionless quantity $\tau=t/R_0$ is introduced. It is rather easy to check that the partial solution of the last equation has the next view

$$h(\tau)=const\cdot\exp\left\{2im\,\mathrm{arctg}\left(th\frac{\tau}{2}\right)\right\}.$$

After all, the partial solution of the wave equation (5) can be written as

$$f_m(\alpha,t)=const\cdot\exp\left\{2im\,\mathrm{arctg}\left(th\frac{t}{2R_0}\right)\pm im\alpha\right\}. \tag{7}$$

It is obviously that the signs «+» and «–» correspond to the positive and negative directions of the phase velocities with respect to α variable increasing.

3. The analysis of the results

Instantaneous wave frequency can be found from the expression (7):

$$\omega=\frac{\partial}{\partial t}\left\{2m\,\mathrm{arctg}\left(\mathrm{th}\frac{t}{2R_0}\right)\pm m\alpha\right\}=2m\frac{1}{1+\mathrm{th}^2\frac{t}{2R_0}}\cdot\frac{1}{\mathrm{ch}^2\frac{t}{2R_0}}\cdot\frac{1}{2R_0}=\frac{m}{R_0}\cdot\frac{1}{\mathrm{ch}^2\frac{t}{2R_0}+\mathrm{sh}^2\frac{t}{2R_0}}=\frac{m}{R_0\,\mathrm{ch}\frac{t}{R_0}}\,.$$

Due to the relation $R_0 \operatorname{ch}\frac{t}{R_0} = R(t)$ we finally have

$$\omega = \frac{m}{R(t)}.$$

Thus the frequency decreases inversely to the size of our system that is associated with the Universe. This result is rather prospective. The non trivial result is consist in the fact that the frequency is discrete and multiple of inversed the Universe radius $1/R$. The frequency step equals to

$$\Delta\omega = \frac{c}{R},$$

where the speed of light c is restored. Due to observational data the radius R has to be greater than 10^4 Mps or 10^{26} m [7]. So the following numeral appreciation is

$$\Delta\omega \le 10^{-18}\ c^{-1}.$$

and this is much smaller than up-to-day experimental possibilities.

The phase velocities for all components equal to c with respect to the associated reference frame. Really,

$$\lambda_m = \frac{2\pi R}{|m|}$$

and it means that

$$\lambda_m \nu_m = \frac{2\pi R}{|m|} \cdot \frac{\omega_m}{2\pi} = c.$$

Thus due to the investigated model the possibility of the University closeness cannot be revealed with observation of discrete background spectrum. At the other hand the model introduces upper restriction on the wave lengths that can have as a sequence some restriction on the energy spectrum of the background radiation.

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Анти-де Ситтер екі өлшемді кеңістігінде толқын теңдеуінің шешімі

Мақалада толқын теңдеуінің шешімі бір қабатты гиперболоидта қарастырылған. Ол толқындардың фазалық жылдамдығы Бүкіләлемдік санау жүйесіне қатысты өсетін екі өлшемді үлгі ретінде берілген, бірақ тең координаттармен салыстырғанда тұрақты күйде қалатындығы көрсетілген. Кеңістік қисығына сәйкес мүмкін жиілік пен толқын сандары дискретті және уақыт өткен сайын пропорционал түрде кішірейе береді. Бұл санауда, тәжірибелік мүмкіндіктерге қарағанда, дискреттілік қадамы көп есе кіші болу керектігі сандық бағалау негізінде айқындалды.

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Решение волнового уравнения в двумерном пространстве анти-де Ситтера

В работе рассмотрено решение волнового уравнения на поверхности однополостного гиперболоида, которая рассматривается как двумерная модель ускоренно расширяющейся замкнутой Вселенной. Показано, что фазовая скорость волн возрастает относительно глобальной системы отсчета, но остается постоянной относительно сопутствующих координат. Возможные частоты и волновые числа дискретны и со временем падают пропорционально уменьшающейся кривизне пространства. На основе числовых оценок показано, что шаг дискретности должен быть много меньше экспериментальных возможностей для его измерения.

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