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Calculation of continuous beams taking into account the elastic compliance of supports

This work considers multispan structures (beams, columns) supported on intermediate elastically compliant supports. The structures are also extensively used in high-rise construction. The research is carried out by an exact analytical force method, based on the equation of five moments. The basic resolving equations are given. The work of a two-span structure with elastically compliant supports is considered in detail. The chart of flexure, the bending moments and the lateral forces are obtained depending on the compliance coefficient α, which varies within the limits of $0, .., \infty$. The analysis of the obtained results is carried out from the point of view of rational work of the operated structures.

Keywords: multi-storey frame columns, power loads, compliance of supports, vertical displacement of beams components, compliance coefficient.

Introduction

When calculating the load-bearing multi-storey frame columns of residential, public and industrial buildings and facilities from the effect of various power loads (including the effect of horizontal wind loads), it becomes necessary to take into account horizontal displacements of the column components at the levels of each of the frame floors, which can substantially change the distribution pattern of internal forces along the length of the columns [1, 2].

The columns design diagram, in the general view, in the presence of a number of intermediate supports, can be displayed as a multi-span (by the number of the building floors) continuous beam on elastic-settling supports (Fig. 1, *a*), («m» is the total number of beam supports).

Such structures are extensively used in high-rise construction; while the compliance of the intermediate supports models the flexibility of the intermediate floors. In this case, it is important to do the research on the compliance of the supports (components) to the strain-stress state of the structures (flexures, internal forces) [3]. Theoretical bases of calculations of multi-span structures (beams and frames) are considered with variable spans, bending stiffness, compliance coefficients, and external load intensity. Obtained diagram dependences can be used in practical design of load-bearing structures of multi-storey buildings.

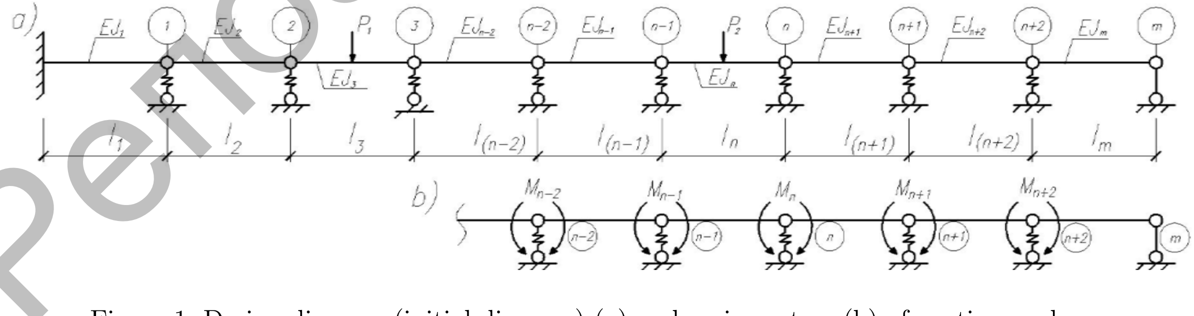

Figure 1. Design diagram (initial diagram) (a) and main system (b) of continuous beam

An elastic support is a support the movement of which is directly proportional to the support reaction arising from the external action.

In practical construction, examples of such supports can serve as long columns, cross-beams of the roadway of various types of bridges, as well as pontoons serving as supports for a floating bridge [4–6].

The elastic supports are characterized by the compliance coefficient C_i is the displacement of support caused by a single force [7].

Purpose and objectives of the research

Due to the fact that the elastic compliance of the components (posts) of building structures significantly affects the strain-stress state of the objects of research, it is important to develop methods for calculating them with a variety of structural concepts, external loading, including the compliance rate of the supports.

In this work, a complex approach has been applied to solve these problems: the difference in the spans' dimensions, bending stiffness, the intensity of uniformly distributed loading, and the variety of the supports compliance coefficients are taken into account simultaneously in the design diagrams.The resolving equations of the force method will take into account the above mentioned factors.

For the proposed theory illustration, a two-span continuous beam was studied in detail, for which dependencies of displacements and internal forces on the compliance coefficients of the 2^{nd} and 3^{rd} supports were obtained. The obtained dependency diagrams will extensively use in the practice of designing the load-bearing building structures.

Theory and calculation methods

The calculation of such structures can be made by an exact analytical force method; The main unknowns in the main system, shown in Fig. 1, b, will be support bending moments $M_1, M_2, \ldots, M_{m-1}$. In this case, the resolving canonical equation of the force method will have the form of an equation of five moments [8]

$$\delta_{n,n-2}M_{n-2} + \delta_{n,n-1}M_{n-1} + \delta_{n,n}M_n + \delta_{n,n+1}M_{n+1} + \delta_{n,n+2}M_{n+2} + \Delta_{np} = 0. \tag{1}$$

The canonical equation coefficients (1) determined by the known procedure of the force method, based on the Vereshchagin rule, will have the form [9]:

$$\begin{aligned}
&\delta_{n,n-2} = \frac{C_{n-1}}{l_{n-1}l_n};\\
&\delta_{n,n-1} = \frac{l_n}{6EJ_n} - \frac{C_{n-1}}{l_n}\left(\frac{1}{l_{n-1}} + \frac{1}{l_n}\right) - \frac{C_n}{l_n}\left(\frac{1}{l_n} + \frac{1}{l_{n+1}}\right);\\
&\delta_{n,n} = \frac{l_n}{3EJ_n} + \frac{l_{n+1}}{3EJ_n l_{n+1}} + \frac{C_{n-1}}{l_n^2} + C_n\left(\frac{1}{l_n} + \frac{1}{l_{n+1}}\right)^2 + \frac{C_{n+1}}{l_{n+1}};\\
&\delta_{n,n+1} = \frac{l_{n+1}}{6EJ_{n+1}} - \frac{C_{n+1}}{l_{n+1}}\left(\frac{1}{l_{n+1}} + \frac{1}{l_{n+2}}\right) - \frac{C_n}{l_{n+1}}\left(\frac{1}{l_n} + \frac{1}{l_{n+1}}\right);\\
&\delta_{n,n+2} = \frac{C_{n+1}}{l_{n+1}l_{n+2}};
\end{aligned} \tag{2}$$

$$\Delta_{1p} = \frac{B_n^f}{EJ_n} + \frac{A_{n+1}^f}{EJ_{n+1}} + \frac{C_{n-1}}{l_n}R_{n-1} - l_n\left(\frac{1}{l_n} + \frac{1}{l_{n+1}}\right)R_n + \frac{C_{n+1}}{l_{n+1}}R_{n+1},$$

where $R_{n-1},\ R_n,\ R_{n+1}$ are respectively the supports reactions $n-1,\ n,\ n+1$ in the main system (Fig. 1, b); $B_n^f,\ {}^f_{n+1}$ are fictitious reactions of the considered support in n and $n+1$ beam spans; for example, under the action of a uniformly distributed load q_i for a span of length l_i, these reactions equals to: $B_i^f = A_i^f = \frac{q_i l_i^3}{24}$.

Let us introduce the following notation:

$$\alpha_i = \frac{C_i C_0 (EJ_0)}{l_0^3}, \tag{3}$$

where $C_0,\ EJ_0,\ l_0$ are respectively scaling (conventional) values of the compliance coefficient, bending stiffness, beam span; C_i is digital coefficient which varies within the limits of $C_i = 0, 1, ..., \infty$; the compliance of the corresponding support is absent at $C_i = 0$ i.e., the rigid support of the beam components takes place; the compliance of the support is very large at $C_i = \infty$, which characterizes the absence of this «i-th» support in the beam design diagram.

In case when all spans have a constant section and are equal-sized ($EJ = \text{const},\ l = \text{const}$), therein the supports have the same compliance coefficient $C_i = C_0 = C = \text{const}$, then by introducing the notation,

$$\alpha_i = \frac{6EJC}{l^3}$$

we obtain instead of (1) a simplified canonical equation of the form

$$\alpha M_{n-z} + (1-4\alpha)M_{n-z} + (4+6\alpha)M_n + (1-4\alpha)M_{n+z} + \alpha M_{n+z=} = -\frac{6EJ}{l}\Delta_{np}.$$

In this case

$$\frac{6EJ}{l}\Delta_{np} = \frac{6}{l}\left(B_n^f + A_{n+1}^f\right) + \alpha l\left(R_{n-1} - 2R_n + R_{n+1}\right).$$

For testing purposes of the proposed calculation theory and researching of the continuous beams or columns work, let us consider the structure in the form of a two-span beam with variable bending stiffness, with different spans and different compliance coefficients of supports loaded with a uniformly distributed load of different intensity on the spans (Fig. 2, *a*).

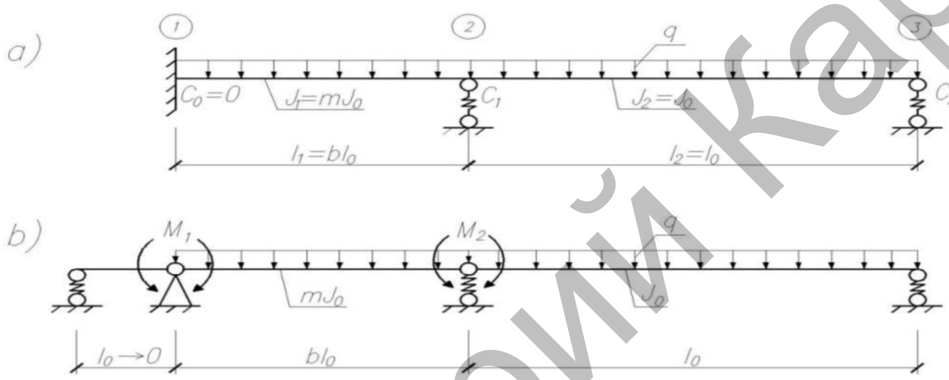

Figure 2. Design diagram of two-span continuous beam (a) and its main system (b)

System of canonical equations according to (1) (at $n = 2$) is as follows:

$$\begin{cases} \delta_{11}M_1 + \delta_{12}M_2 + \Delta_{1p} = 0, \\ \delta_{21}M_1 + \delta_{22}M_2 + \Delta_{2p} = 0. \end{cases} \tag{4}$$

According to the formula (2), we determine the coefficients of equation (4) taking into account the expression (3):

$$\delta_{11} = \frac{l_0}{EJ_0}\left(\frac{b}{3m} + \frac{C_1}{b_n^2}\alpha\right); \quad \alpha = \frac{C_0 EJ_0}{l_0^3};$$

$$\delta_{12} = \delta_{21} = \frac{l_0}{EJ_0}\left(\frac{b}{6m} - \frac{C_1(1+b)}{b^2}\alpha\right);$$

$$\delta_{22} = \frac{l_0}{EJ_0}\left[\left(\frac{b}{3m} - \frac{1}{3}\right) + \frac{\alpha}{b^2}\left(C_1\left(1+2b+b^2\right) + C_2 b^2\right)\right]; \tag{5}$$

$$\Delta_{1p} = \frac{ql_0^3}{EJ_0}\left(\frac{0.0417b^3}{m} + 0.5C_1\alpha\frac{(1+b)}{b}\right);$$

$$\Delta_{2p} = \frac{ql_0^3}{EJ_0}\left\{\left(\frac{0.0417b^3}{m} + 0.0417\right) - \left(\frac{1+b}{b}\right)\alpha\left[0.5C_1(1+b) + 0.5C_2\right]\right\}.$$

Results and Discussion

As a test task representing the proposed theory, let us consider an example of calculating of two-span continuous beam with identical spans, constant bending stiffness, and equal values of the compliance supports coefficients 2, 3 (see Fig. 3, *a*) i.e.:

$$l_1 = l_2 = l_0 = 6\text{m};\ \ C_1 = C_2 = C_0;\ \ EJ_1 = EJ_2 = EJ_0;\ \ q = \text{const} = q_0;\ \ \alpha = \frac{C_0 EJ_0}{l_0^3}.$$

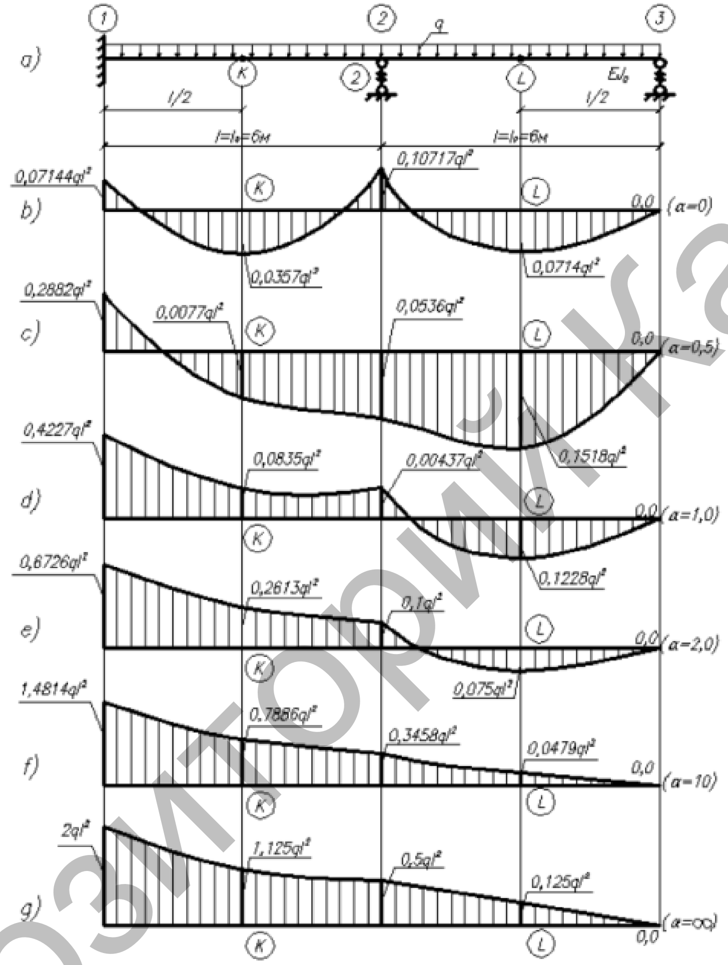

Figure 3. Bending moment diagrams under a variety of «*a*» values

According to the formula (5) we have:

$$\delta_{11} = \left(\frac{0.33+\alpha}{EJ_0}\right) l_0;\ \ \delta_{12} = \delta_{21} = \frac{l_0}{EJ_0}(0.167 - 2\alpha);\ \ \delta_{22} = \frac{l_0}{EJ_0}(0.667 + 5\alpha); \qquad (6)$$

$$\Delta_{1p} = \frac{q_0 l_0^3}{EJ_0}(0.0417 + \alpha) = b_1;\ \ \Delta_{2p} = \frac{q l_0^3}{EJ_0}(0.0834 - 1.5\alpha) = b_2.$$

Solving the system of linear algebraic equations of the second order (taking into account the expressions (6), we determine the support moments (Fig. 2, b), (in general form):

$$M_1 = \frac{\left(-b_1 - \delta_{12}\left(\frac{b_2\delta_{11}}{\delta_{12}} - b_1\right)\right) \Big/ \left(\delta_{12} - \frac{\delta_{11}\delta_{22}}{\delta_{21}}\right)}{\delta_{11}};\ \ M_2 = \frac{b_2\left(\frac{\delta_{11}}{\delta_{21}} - b_1\right)}{\delta_{12} - \frac{\delta_{11}\delta_{22}}{\delta_{12}}}. \qquad (7)$$

According to the expression (7), taking into account the initial data of the task, we have:

$$M_1 = k_1 q_0 l_0^2; \ \ M_2 = k_2 q_0 l_0^2, \tag{8}$$

where

$$k_1 = \frac{-1 - 75\alpha - 99\alpha^2 - 324\alpha^3}{14 + 258\alpha + 720\alpha^2 + 216\alpha^3}; \ \ k_2 = \frac{0.75 - 0.18\alpha + 18\alpha^2}{-7 - 108\alpha - 36\alpha^2}. \tag{9}$$

Table shows the results of the calculation of the continuous beam (Fig. 3, *a*), produced by the formulas (8), (9) with the following data:

$$l_1 = l_2 = l; \quad EJ_1 = EJ_2 = EJ; \quad q = \text{const}; \quad \alpha = 0.0; \, 0.5; \, 0.1; \, 1.0; \, 2.0; \, 10.0; \, \infty.$$

Figure 3 b–g shows the diagrams of bending moments at different values of the coefficients «α_i», taking into account the effect of the variable support coefficient C_0.

Figures 4–7 show, respectively, the dependence diagrams of bending moments M_i, flexures C_i, lateral forces Q_i, support reactions R_i of the beam depending on the coefficient value α.

The analysis of the dependencies in Figures 4–7 shows the following:

1. The ordinate values of moment diagram (Fig. 4) depend essentially on the value α and change their signs (from the two-digit torque value to single-digit (negative) torque values) at $\alpha \geq 2.0$.

2. The bending moments, the lateral forces, the reactions in the beam sections are increasing monotonically with a monotonous increase of α, except the support reaction at component 3, which in this case is decreasing monotonically.

3. The values M_1, M_2 take positive value in the $\alpha \geq 1.0$ values range (Fig. 4).

4. The flexures values are increasing slowly in the beam sections at $\alpha \leq 1.0$ and increasing rapidly at $\alpha \geq 2.0$ (Fig. 5), in this case the increase of elastic compliance is particularly significant.

5. The dependency diagrams on the value of lateral forces Q_1, Q_2^{left} and Q_2^{right}, Q_3 are parallel in their lineament (Fig. 6).

6. The dependency diagrams of reactions R_1 is increasing monotonically with an increase of α, therein the R_2 reaction is decreasing monotonically, and the R_3 reaction diagram has a complex lineament at $\alpha \leq 2.5$, and it is decreasing monotonically at $\alpha \geq 2.5$ (Fig. 7).

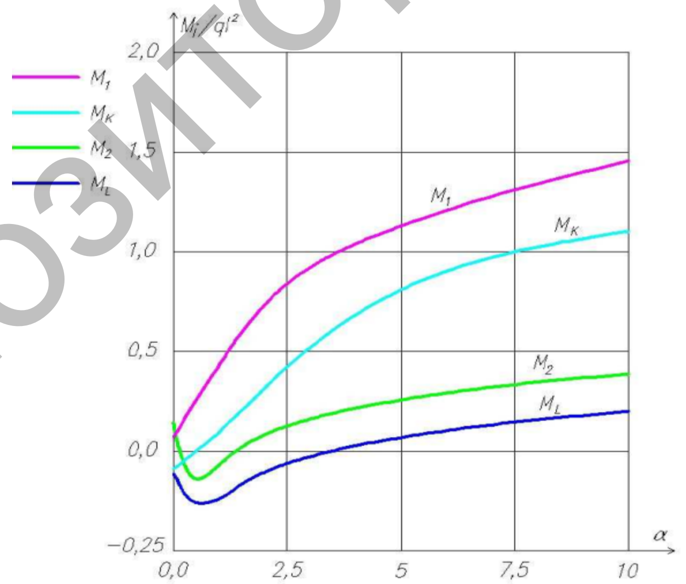

Figure 4. Dependence of the bending moment values M_K, M_L, M_1, M_2 on the coefficient of elastic compliance of the beam supports «α»

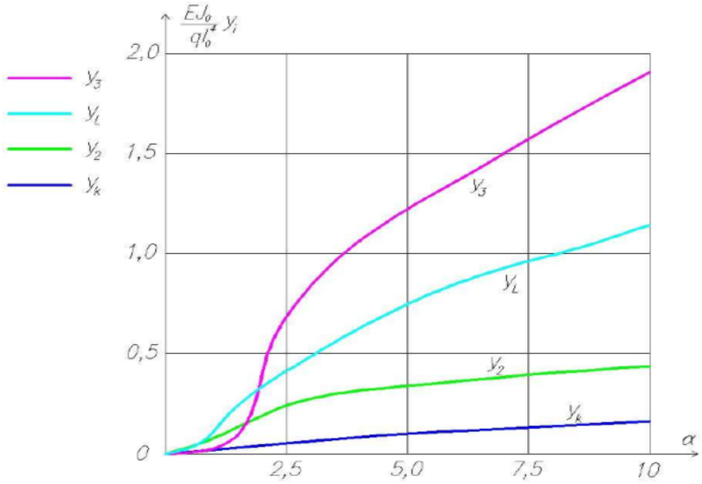

Figure 5. Dependence of flexure on «α»

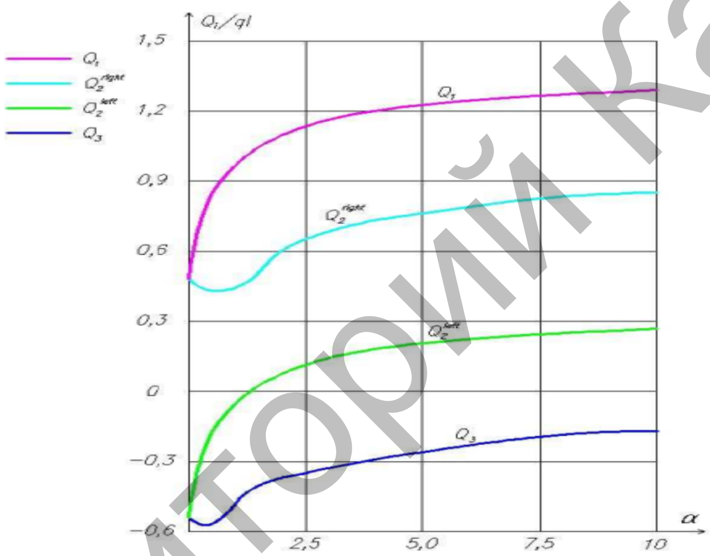

Figure 6. Dependence of lateral forces on «α»

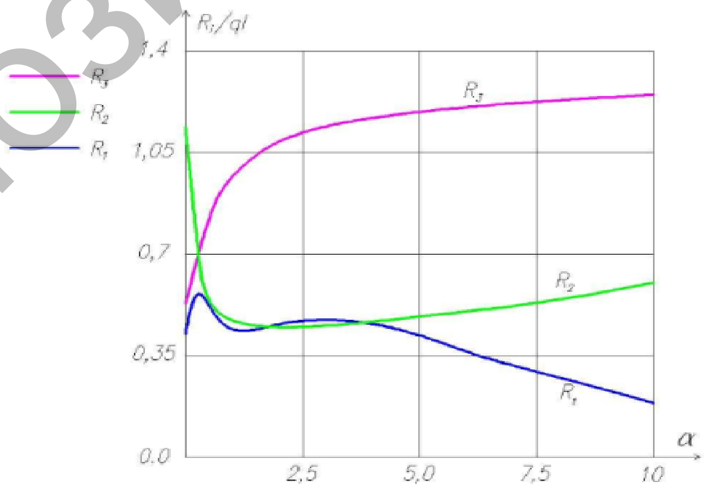

Figure 7. Dependence of support reaction on «α»

The results obtained in this work, for a two-span continuous beam based on the equation of five moments in terms of their reliability, are estimated as follows:

1) Relative comparison of results:

The bending moment diagrams (Fig. 3) with an increase of the compliance coefficient of the 2^{nd} and 3^{rd} beam supports «α» from a two-digit lineament (Fig. 3, b) change to a single-digit negative lineament (Fig. 3, g), which reflects the actual work of the structure at the action of the load uniformly distributed on the spans (Fig. 3, *a*); the indicated dependence is also present in the ordinates values of the moment diagram (see Table);

Table

Dependence of internal forces and support reactions of the beam on the value «α»

α	0.0	0.5	1.0	2.0	10.0	∞
$\frac{M_1}{ql^2}$	-0.0074	-0.2882	-0.4127	-1.6726	-1.4814	-2.0
$\frac{M_2}{ql^2}$	-0.10717	0.0536	-0.000497	-0.100	-0.3458	-0.500
$\frac{M_3}{ql^2}$	0.00	0.00	0.00	0.00	0.00	0.00
$\frac{Q_1}{ql}$	0.4643	0.8428	0.90833	1.0726	1.2583	2.0
$\frac{Q_2^{\text{left}}}{ql}$	-0.535	-0.1582	-0.0917	0.0726	0.2583	1.0
$\frac{Q_2^{\text{right}}}{ql}$	0.6072	0.4464	0.5044	0.600	0.8458	1.0
$\frac{Q_3}{ql}$	-0.3928	-0.5538	-0.4956	-0.400	-0.1542	0.0
$\frac{R_1}{ql}$	0.4643	0.8428	0.9083	1.073	1.2583	2.0
$\frac{R_2}{ql}$	1.1423	0.6046	0.5236	0,5274	0,5875	0.0
$\frac{R_3}{ql}$	0.3928	0.5536	0.4956	0.400	0.1542	0.0

2) Absolute comparison of the results:

a) At $\alpha = 0$ (the compliance of the supports is minimal and there is a rigid support of the 2^{nd} and 3^{rd} supports) (Fig. 3, b) the values of the support ordinates of the moment diagram coincide (without the presence of an error) with the calculation of a beam according to the well-known theory of three moments [7, 9];

b) At $\alpha = 0$ (the compliance of the supports is maximal and there is a single-span cantilever beam with a length «$2l$») (Fig. 3, *a*) the greatest bending moment in the support «1» is equal to ($M_1 = 2ql^2$), (Fig. 3, g) which coincides with the known value for the cantilever beams $\left[M_1 = \dfrac{q(2l)^2}{2} = 2ql^2\right]$.

Other results (at $\alpha \neq 0, \ \alpha \neq \infty$), given in this work are original, i.e., new in the scientific literature for today.

As for comparing the obtained results with the experimental data of other authors, we have not found similar results for today, apparently, the experiments on the subject of our research have not yet been carried out.

Along with this, it should be noted that we compared the results with analytical calculations: the relative (semantic) and absolute (the coincidence of data with the results obtained by other methods) comparison of the results (see the section «analysis of dependencies»).

Conclusion

1. In this work, the research is performed on the stress-strain state of a two-span continuous beam with elastic-settling supports for equal spans, with the same bending stiffness under the action of a uniformly distributed load of the same intensity.

2. The method of calculation is an exact analytical force method based on the five moments equation [9], [10], which is used here to calculate a variety of continuous beams (columns) with multiple spans, with different spans values, variability of bending stiffness and lateral uniformly distributed load on the spans, and also at different values of the compliance coefficients of the beams supports (Fig. 2, *a*).

3. The two-span continuous beam of constant bending stiffness, equal to spans and loads on them, as well as the same parameters of C_1, C_2^0 was performed as an illustration of the proposed theory operability (Fig. 3, *a*); the flexure dependency diagrams, the bending moments and the lateral forces on the change in the compliance coefficients C_1, C_2 were obtained for this variant (Fig. 4–7).

4. At the same time it is established that with an increase in the compliance of the supports, the lineaments of the diagrams M, Q change substantially and the beam flexures increase therein.

5. The proposed theory also makes it possible to calculate the columns of multi-storey buildings, the design diagram of which can be represented as a multi-span continuous beam. The presence of intermediate floorings, in this case, will be modeled as the elastic compliance of the components of the columns joints and flooring.

6. The proposed theory and the practical results can be used both in scientific researches, and in practice of designing of load-bearing structures of high-rise and unique buildings.

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Ж.С. Нугужинов, С.К. Ахмедиев, С.Р. Жолмағамбетов, О. Хабидолда

Тіректердің серпімді икемділігін ескерумен кесілмейтін арқалықтарды есептеу

Мақалада аралық серпімді икемді тіректерге сүйенген көпаралықты конструкцияларды (арқалықтарды, бағаналарды) зерттеу орындалған. Сонымен қатар конструкциялар биік құрылыста кең қолданыс тапқан, зерттеу бес сәттің теңдеуінің негізінде дәл талдамалы күштер әдісімен келтірілген. Негізгі бұзушы теңдеулер берілген. Серпімді икемді тіректермен қосаралықты конструкцияның жұмысы толық қарастырылған. $0,..,\infty$ шегінде өзгеретін икемділік коэффициентіне α тәуелділікте иілулердің, иілу кездері мен көлденең күштерінің кестелері алынған. Пайдаланылатын конструкциялардың дұрыс жұмысы тұрғысынан алынған нәтижелерді талдау жүргізілген.

Кілт сөздер: көп қабатты рамалы қаңқалардың бағаналары, күштік жүктемелер, тіректердің икемділігі, арқалықтардың тораптардың тік ығысулары, икемділік коэффициенті.

Ж.С. Нугужинов, С.К. Ахмедиев, С.Р. Жолмагамбетов, О. Хабидолда

Расчет неразрезных балок с учетом упругой податливости опор

В статье выполнено исследование многопролетных конструкций (балок, колонн), опертых на промежуточные упруго податливые опоры. Такие конструкции находят широкое применение в высотном строительстве. Исследование проведено точным аналитическим методом сил, на основе уравнения пяти моментов. Приведены основные разрешающие уравнения. Подробно рассмотрена работа двухпролетной конструкции с упруго податливыми опорами. Получены графики прогибов, изгибающих моментов и поперечных сил в зависимости от коэффициента податливости α, который меняется в пределах 0...∞. Проведен анализ полученных результатов с точки зрения рациональной работы эксплуатируемых конструкций.

Ключевые слова: колонны многоэтажных рамных каркасов, силовые нагрузки, податливость опор, вертикальные смещения узлов балок, коэффициент податливости.

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