UDC 372.8

G.A. Yessenbayeva[1], D.N. Yesbayeva[2], K.S. Kutimov[1], D.A-H. Nurgali[1], T. Kh. Makazhanova[1]

[1]Ye.A. Buketov Karaganda State University, Kazakhstan;
[2]Shanghai Factory-Amigo EC Technology Co., Ltd, China
(E-mail:esenbaevagulsima@mail.ru)

On one approach to solving mechanical problems of kinematics and dynamics

This article is devoted to the application question to the mechanics of one of the solving problems methods used in the exact sciences. In the article the structure of the given method is presented, its main components are highlighted and characterized, the moments on which it is necessary to pay attention are marked and the main stages of solving the mechanics problems are described. The considered approach to solving problems in exact sciences is, generally speaking, classical, but the application of this method to a concrete subject area entails the specific features of the practical application, which are noted in this article. The researching approach of solving problems is demonstrated for mechanical problems in kinematics and dynamics. The main attention is focused on the key stages of this method. Theoretical material on kinematics and dynamics is specially selected and rationally formulated in the necessary brief scope for solving problems. In the classification of problems of kinematics and dynamics, their main types are given. The schemes for solving the problems of kinematics and dynamics are formulated in the most general form, but the notes give explanations of the use of the presented solution schemes for particular cases. The article is focused mainly on mechanics, physicists and students in technical specialties.

Keywords: kinematics, dynamics, forces, equations of motion, the main types of problems of kinematics and dynamics, schemes for solving the problems of kinematics and dynamics.

A teaching method comprises the principles and methods used by teachers to enable student learning. These strategies are determined partly on subject matter to be taught and partly by the nature of the learner. For a particular teaching method to be appropriate and efficient suggestions to design and selection of teaching methods must take into account the nature of the subject matter. It should be remembered that the teacher's primary role is to coach and facilitate student learning and overall comprehension of material.

The main task of the modern teacher of exact sciences is not to create mechanical application of theoretical knowledge and formulas for students and master students, but to create a deep understanding of the essence of the problem, the ability to navigate sensibly in mathematical models, algorithms, schemes and the ability to use them in solving a particular problem.

Mechanics is a large area of science, which is divided into sections: classical mechanics, relativistic mechanics and quantum mechanics. In addition, mechanics includes some other theories, such as the theory of oscillations, the theory of elasticity, theory of stability, mechanics of liquids and gases etc.

Classical mechanics, in turn, is divided into statics (the study of the equilibrium of bodies), kinematics (the study of the motion of bodies without consideration of reasons) and dynamics (the study of the motion of bodies).

As experience of teaching has shown, this technique, used by many teachers, has many advantages. It is a a clear structure, flexible and perspective nature of presentation, accessibility for understanding, good learning outcomes.

Mechanics problems are solved in several stages: first, draw a diagram of the object or objects. Plans should display all the physical characteristics of the system: speed, acceleration, time, distance, force application, etc. in vector form, i.e. clearly indicate which laws should be used to find the result. Step two is to record all the laws of motion and equations. Solve these equations; add dimension and you get the result [1].

Regardless of which of the methods of mechanical problem solving is taught, three basic concepts should be attended to. These concepts are:

1) necessary prerequisite knowledge (the theoretical material),
2) systems approach (knowledge of basic types of problems),
3) a model for problem solving (a scheme of problem solving).

It's important to have a perfect understanding of the concept involved before you attack a problem. It is a known fact that human advancement comes through reasoning, therefore knowledge of the theoretical material is necessary. Theoretical material should not be a lecture. It should contain only the concepts and formulas necessary for solving problems.

Let us present the embodiment of this teaching method using the example for solving of problems in kinematics and dynamics of a material point and the simplest systems.

Here, for solving the problems of kinematics and dynamics, theoretical material is specially adapted and rationally formulated in a brief volume. Having given theoretical information, after determining the type of the problem being solved and choosing the calculation scheme, the student or the master student can begin to solve his problem.

1. The theoretical material

a) Kinematics of a material point and the simplest systems

Kinematics is a branch of classical mechanics that describes the motion of points, bodies (objects), and systems of bodies (groups of objects) without considering the mass of each or the forces that caused the motion.

Kinematics, as a field of study, is often referred to as the «geometry of motion» and is occasionally seen as a branch of mathematics.

A kinematics problem begins by describing the geometry of the system and declaring the initial conditions of any known values of position, velocity and/or acceleration of points within the system. Then, using arguments from geometry, the position, velocity and acceleration of any unknown parts of the system can be determined. The study of how forces act on masses falls within kinetics.

Kinematics is used in astrophysics to describe the motion of celestial bodies and collections of such bodies. In mechanical engineering, robotics, and biomechanics kinematics is used to describe the motion of systems composed of joined parts (multi-link systems) such as an engine, a robotic arm or the human skeleton.

Geometric transformations, also called rigid transformations, are used to describe the movement of components in a mechanical system, simplifying the derivation of the equations of motion. They are also central to dynamic analysis.

Kinematic analysis is the process of measuring the kinematic quantities used to describe motion. In engineering, for instance, kinematic analysis may be used to find the range of movement for a given mechanism, and working in reverse, using kinematic synthesis to design a mechanism for a desired range of motion. In addition, kinematics applies algebraic geometry to the study of the mechanical advantage of a mechanical system or mechanism [2].

In other words, *kinematics* is a branch of classical mechanics that studies the mathematical description (by means of geometry, algebra, mathematical analysis, etc.) of the motion of idealized bodies (material point, absolutely solid body, systems of bodies), without considering the causes of motion (mass, forces, etc.). The initial concepts of kinematics are space and time.

A physical quantity is a quantitative characteristic of the property of material objects or phenomena (processes). Each physical quantity is established by a single-valued method of its measuring. Its measuring is experimental determination or calculation. The definition of a physical quantity indicates the principal way of measuring it. In other words, *a physical quantity* is a physical property that can be quantified. This means it can be measured and/or calculated and expressed in numbers.

A material point is a physical concept (model, abstraction) representing a body whose dimensions (and shape) can be neglected under the conditions of the given problem.

The body of the reference is the body relative to which the motion of other bodies is considered.

The frame of reference is the set of the coordinate system associated with the reference body and the set of synchronized clocks which are placed at different points of the coordinate system.

The position of a point in space is the most fundamental idea in particle kinematics. To specify the position of a point, one must specify three things: the reference point (often called the origin), distance from the reference point and the direction in space of the straight line from the reference point to the particle. Exclusion of any of these three parameters renders the description of position incomplete.

The position of a material point relative to a given frame of reference (in a frame of reference) S is given by its coordinates or the radius vector r.

The radius vector r of the material point with respect to the given frame of reference is a vector whose origin is the coordinates beginning of this system and the end is located at the material point.

Every motion of the body can be decomposed into two main types of motion: translational motion and rotational motion.

Translational motion is the movement of the body, in which all its points move along the same way.

Rotational motion is the movement of the body, in which all its points move along circles whose centers lie on the same straight line. This line is called *the axis of rotation*.

The law of motion of a material point with respect to a given reference frame is the dependence of the radius vector or the coordinates of the material point on time.

Relativity of movement is the dependence of a certain motion trajectory, a certain path, speed and displacement of some body from the selected frame of reference.

The trajectory of the motion of a material point is a line, described in space by the end of the radius vector of a material point.

The trajectory equation is given by the complex of two equations that can be obtained by excluding the time from the law of motion in coordinate form. Note that the law of motion in the coordinate form is the equation of the trajectory, given in a parametric form.

The displacement of the material point $\Delta r(t)$ is the change in the radius vector of the material point from the time moment t to the time moment $t + \Delta t$.

The path $s(t)$ is the distance passed the material point along the trajectory (length of the trajectory) during the time t.

The velocity of a material point v relative to a given coordinate system is a physical quantity which is equal to the derivative of the radius vector of the material point with respect to time.

The initial conditions for a material point are the values of the radius vector and the velocity at the initial moment of time t_0 with respect to the given coordinate system.

Acceleration of a material point relative to a given coordinate system is a physical quantity which is equal to the derivative of the velocity of a material point with respect to time.

Tangential acceleration a_τ is the acceleration component along the tangential direction of the velocity.

The normal acceleration a_n is the acceleration component which is a perpendicular to the direction of the velocity.

Angular velocity is a vector quantity characterizing the speed of rotation of the body.

Angular acceleration is a quantity characterizing the rapidity of the change in the angular velocity.

A mechanical system is a collection of material bodies.

The system of material points is a collection of bodies, each of which can be considered as a material point. Further, we will assume that any mechanical system considered by us can be regarded as a system of material points.

An absolutely solid body is a body (a system of material points), the distances between any two material points of which do not change under the conditions of the given problem.

The methods and dependencies of kinematics are used in kinematic studies of movements, in particular, in the calculation of the motions transmission in various constructions, mechanisms, machines, etc.

b) Dynamics of a material point and the simplest systems

Dynamics is the science of how things move. A complete solution to the motion of a system means that we know the coordinates of all its constituent particles as functions of time. For a single point particle moving in three-dimensional space, this means we want to know its position vector $r(t)$ as a function of time. If there are many particles, the motion is described by a set of functions $r_i(t)$, where i labels which particle we are talking about. So generally speaking, solving for the motion means being able to predict where a particle will be at any given instant of time. Of course, knowing the function $r_i(t)$ means we can take its derivative and obtain the velocity $v_i(t) = \frac{dr_i}{dt}$ at any time as well.

Position is usually described by mathematical quantities that have all these three attributes: the most common are vectors and complex numbers. Usually, only vectors are used.

The complete motion for a system is not given to us outright, but rather is encoded in a set of differential equations, called *the equations of motion* [3].

In other words, *dynamics* is a branch of classical mechanics in which the causes of the emergence of mechanical motion are studied. Dynamics operates with such concepts as mass, force, momentum, momentum, energy.

Dynamics is applied in different sciences: in physics, astronomy, the sciences of the Earth, engineering, chemistry, biology, music, etc. Classical dynamics is based on the laws of Newton.

Newton's Laws of motion

Aristotle held that objects move because they are somehow impelled to seek out their natural state. Thus, a rock falls because rocks belong on the earth, and flames rise because fire belongs in the heavens. To paraphrase Wolfgang Pauli, such notions are so vague as to be «not even wrong». It was only with the publication of Newton's Principia in 1687 that a theory of motion which had detailed predictive power was developed.

Newton attempted to explain how objects move using as few assumptions as possible. These assumptions are what we call Newton's Laws today. They are remarkable in that they have stood the test of time for almost all motion except those at the smallest scales (quantum mechanics) and the largest scales (general relativity). Even then, we have mostly just added or modified some assumptions that we had thought reasonable to assume about the nature of the universe, but the three laws have mostly remained. The laws are used to deduce the fundamental equation we will be using to study almost all of classical physics below. The wording of the laws has been altered slightly in order to be less jarring to a modern student.

Newton's Laws allow us to write the equation of motion for any mechanical system, if you know the power of interaction. A total of three: the law of inertia (maintaining speed body), the law of motion and the law of the pair interaction [3].

Newton's First Law (The Law of Inertia): There are such reference frames with respect to which the isolated material point (on which forces do not act) moves uniformly and rectilinearly or is at rest. Such frames of reference are called *inertial frames*.

At this point, Newton has not told us what a force is, so this law isn't a description of how bodies move. You may think «I know what a force is, I force something to move when I push on it. «However, that» contact force» is not the only type of force Newton means to deal with. He has basically used this statement to define what a force is: anything that changes the linear motion (including zero motion) of a body. This allows us to include gravity as a force. As a matter of fact, we will see that classical physics successfully describes all forces in terms of fields (similar to the way gravity works). Your pushing on an object is actually a result of electromagnetic field interactions between the particles in your hand and the particles in whatever you're pushing. Your hand never actually comes into direct contact with anything, in the sense of there being no space between your hand and another object.

Newton's Second Law (The Law of Acceleration): In the inertial reference frame, the product of the mass of a material point by its acceleration is equal to the sum of all forces acting on this material point from the side of other bodies: $ma = \sum_i F_i$.

This law is extremely rich in content, as Newton used devices similar to what we refer to today as Newton's cradle to note that the «quantity of motion» given to the end ball (assuming perfect collisions where all the «motive force» of the ball you lift on one end is transferred to the ball at the other end) is proportional to the final velocity of the ball you initiate. He also noted that different types of balls imparted larger arcs to other balls and thus he came up with the product of mass (a quantity proportional to the object's weight) and velocity as being the «quantity of motion» of an object that should be preserved in perfect collisions. Today, we call this product the *momentum* of an object and usually denote it as $p = mv$, where p and v have both a magnitude and direction, as described by Newton's laws. Manipulating objects that have both magnitude and direction (called vectors) is covered by vector algebra, which will be talked about in a supplementary review guide. Now, if we denote the force vector by $\bar{F}$, then Newton's second law tells us that $\bar{F} = \frac{d\bar{p}}{dt} = \frac{d}{dt}\left(m\bar{v}\right) = m\frac{d\bar{v}}{dt}$. The $\frac{d}{dt}$ is just the ordinary time derivative from calculus. If you have not yet studied calculus, the above can also be written as $\bar{F} = m\bar{a}$, where a is the change in velocity with respect to time, also known as acceleration.

Newton's Third Law (The Law of Interaction): To every action there is always opposed an equal reaction; or the mutual actions of two bodies upon each other are always equal, and directed to contrary parts.

Main point of Newton's third law: action = reaction.

This law is primarily for idealized objects such as spheres and points that act on each other's centers of motion. However, it is the impetus that allowed us to separate mass from weight. In symbolic terms, it states that if one body exerts a force $\bar{F}_1$ on another body, then the second body exerts a force $\bar{F}_2$ on the first body, and in vector terms $\bar{F}_1 = -\bar{F}_2$. Using the equation we derived from the second law, it is not hard to see that the mass can then be measured by defining one mass to be a standard, and using the ratio of accelerations as the mass of the second object [4].

Newton's three Laws of Motion may be stated as follows:

I. A body remains in uniform motion unless acted on by a force.

II. Force equals rate of change of momentum: $\overline{F} = \frac{d\overline{p}}{dt}$.

III. Any two bodies exert equal and opposite forces on each other.

In other words, the third law of Newton is formulated as follows: The forces of interaction of two material points:

1) paired and attached to different material points,
2) of the same nature,
3) are equal in absolute value,
4) are opposite in direction,
5) are directed along a straight line connecting material points.

The laws of dynamics are Newton's laws and laws that describe the individual properties of forces.

The laws that describe the individual properties of forces

1) *Gravitational forces*

The law of universal gravitation: Material points are attracted to each other with forces F_{ij} and F_{ji}, modules which are proportional to the product of their masses and inversely proportional to the square of the distance between them: $\overline{F}_{ij} = G\frac{m_i m_j}{r_{ij}^3}\overline{r}_{ij}$, where G is the gravitational constant or the *Cavendish constant* (first measured by Henry Cavendish in 1798), $\overline{r}_{ij} = \overline{r}_i - \overline{r}_j$.

Notice Newton's Third Law in action: $\overline{F}_{ij} + \overline{F}_{ji} = 0$. Now a very important and special feature of this «inverse square law» force is that a spherically symmetric mass distribution has the same force on an external body as it would if all its mass were concentrated at its center. Thus, for a particle of mass m near the surface of the earth, we can take $m_i = m$ and $m_j = M_e$, with $\overline{r}_i - \overline{r}_j = R_e\hat{r}$ and obtain

$$F = -mg\hat{r} = -m\overline{g},$$

where $\hat{r}$ is a radial unit vector pointing from the earth's center and g is the acceleration due to gravity at the earth's surface. Newton's Second Law now says that $a = -g$, i.e. objects accelerate as they fall to earth. However, it is not a *priori* clear why the *inertial mass* which enters into the definition of momentum should be the same as the *gravitational mass* which enters into the force law. Suppose, for instance, that the gravitational mass took a different value, m′. In this case, Newton's Second Law would predict $a = -\frac{m'}{m}$ and unless the ratio $\frac{m'}{m}$ were the *same number* for *all* objects, then bodies would fall with *different accelerations*. The experimental fact that bodies in a vacuum fall to earth at the same rate demonstrates the equivalence of inertial and gravitational mass, i.e. $m' = m$.

The forces of the gravitational interaction of spherically symmetric bodies, as can be easily shown, are determined by the same formula, in which *r* is the radius vector of the second body center relative to the first body center.

The force of gravity acting on the material point is the sum of the force of the gravitational attraction of the Earth (or any other cosmic object) and the centrifugal inertia force acting on the material point in the reference frame connected with the Earth.

The force of gravity acting on the body is the sum of the forces of gravity acting on the material points of this body. In a homogeneous field near the Earth's surface, the gravity force F_τ is equal to the product of the body mass m by the acceleration of the center of the body mass with free fall g (acceleration of gravity) relative to the Earth: $F_\tau = mg$.

Body weight is the force with which the body in the field of gravity acts on a fixed support relative to it or a suspension that prevents the free fall of the body.

2) *Elastic forces*

Elasticity is the ability of a body to resist a distorting influence or deforming force and to return to its original size and shape when that influence or force is removed. Solid objects will deform when adequate forces are applied on them. If the material is elastic, the object will return to its initial shape and size when these forces are removed.

If after termination of the external impact of deformed body recovers its shape and its sizes, the deformation is called *elastic*.

Hooke's Law: For small elastic deformations, the magnitude of the deformation is proportional to the magnitude of the force that causes it.

In particular, when the distension (compression) strain of the elastic rod (spring, rubber cord), the deformation of the rod is proportional to the magnitude of the force acting along the rod: $\Delta l = \frac{1}{k}F$. Here k is the rigidity (elasticity) coefficient of the rod, $\Delta l = l - l_0$ is the elongation of the rod, and l and l_0 are the lengthes of the rod in deformed and undeformed conditions. The law is named after 17th-century British physicist Robert Hooke.

If the force acting on the rod is directed opposite to the indicated direction, then the elastic rod experiences compression. In this case $\Delta l < 0$ in the last formula, we should also consider F as the projection of the force on the X axis of the given coordinate system.

When homogeneous elastic rod with a constant cross section along the rod is deformed (the distension or compression), the relative elongation ε of the rod is proportional to the normal stress σ: $\varepsilon = \frac{1}{E}\sigma$.

Here E is the *Young's modulus* of the material from which the rod is made, $\varepsilon = \frac{\Delta l}{l_0}$ is the *relative elongation* of the rod, $\sigma = \frac{F}{S}$ is the *normal stress*, S is the cross-sectional area of the rod.

We note that for a homogeneous elastic rod with a constant cross-section along the rod, the coefficient of rigidity (elasticity) of this rod is connected with the Young's modulus by the relation: $k = \frac{S}{l}E$.

In the case of distension (compression) of the rod, its transverse sizes decrease (increase). The ratio of the relative transverse compression of the rod to its relative elongation depends only on the material of the rod. This ratio is called *the Poisson coefficient*: $\mu = -\frac{\varepsilon_\perp}{\varepsilon}$.

Here, μ is Poisson's coefficient, $\varepsilon_\perp = \frac{d - d_0}{d_0} = \frac{\Delta d}{d_0}$ is the relative change in the transverse sizes of the rod, d and d_0 are the transverse linear sizes of the rod in the deformed and undeformed states.

When the rod is deformed, there are *internal elastic forces* $F_{упр}$ acting between its parts, which tend to return the rod to an undeformed state. The stress of elastic forces is equal to $\sigma_{упр} = \frac{F_{упр}}{S}$.

Consider a layer of a rod with the boundaries coordinates x and $x + dx$ along the rod. As a result of the action of internal elastic forces, displacements of the left $\xi(x)$ and right $\xi(x+dx)$ borders of the selected layer appear. Then the relative lengthwise deformation ε of this layer equals

$$\varepsilon = \frac{\xi(x+dx) - \xi(x)}{dx} = \frac{\partial \xi}{\partial x} = \xi'_x .$$

In this case Hooke's law takes the form: $\sigma_{упр}(x) = E\varepsilon = E\xi'_x$.

In the case of deformation of the layer, its transverse sizes change. The ratio of transverse deformation to lengthwise deformation is determined by the Poisson's coefficient in accordance with the formula for μ. With accelerated motion of the rod under the action of an external force that causes its deformation, the stresses of elastic forces, that are inhomogeneous along the rod, arise. In this case, the resulting inhomogeneous deformations are, as before, determined by the expressions for μ and $\sigma_{упр}(x)$.

3) ***Friction forces***

Friction (friction interaction) is the process of interaction of bodies during their relative movement (displacement) or when the body moves in a gaseous or liquid medium.

Friction is usually divided into the following types:

– *Dry friction* is friction, when the interacting solids are not separated by any additional layers / lubricants (including solid lubricants). In other words, *dry friction* is a force that opposes the relative lateral motion of two solid surfaces in contact. Dry friction is subdivided into static friction («stiction») between non-moving surfaces, and kinetic friction between moving surfaces. With the exception of atomic or molecular friction, dry friction generally arises from the interaction of surface features, known as asperities. The dry friction is a very rare case in practice. The characteristic distinctive feature of dry friction is the presence of a significant friction force of rest.

– *Boundary friction* is friction, when in the contact area can contain layers and areas of different nature (oxide films, liquid, etc.). The boundary friction is the most common case in sliding friction.

– *Fluid (viscous) friction*, arising from the interaction of bodies separated by a layer of solid (graphite powder), liquid or gas (lubricant) of different thickness, as a rule, occurs during rolling friction, when solid bodies are immersed in a liquid. The value of viscous friction is characterized by the viscosity of the medium. Fluid friction describes the friction between layers of a viscous fluid that are moving relative to each other.

– *Mixed friction* is friction when the contact area contains areas of dry and liquid friction.

– *Elastohydrodynamic (viscoelastic) friction* is the friction, when the internal friction in the lubricating material is of decisive importance. Elastohydrodynamic (viscoelastic) friction occurs when the relative velocities of displacement increase.

– *Lubricated friction* is a particular type of fluid friction when a lubricant fluid separates two solid surfaces.

– *Skin friction* is a component of drag, the force resisting the motion of a fluid across the surface of a body.

– *Internal friction* is the force resisting motion between the elements making up a solid material while it undergoes deformation.

The friction force is a component of the force of direct interaction of bodies when they touch along the plane of contact.

The force of normal pressure (the reaction of the support) is the component of the force of interaction of the bodies with direct contact along the direction of the normal to the plane of contact. In other words, the force of normal pressure (reaction of the support) is the reaction force/forces that are attributed to a support for the system, i.e. if the system is a chair, and the external inputs that exert significant forces on the chair are the Earth (gravitational force) and the floor («Normal» force; Normal here is being used to describe a force perpendicular to the surface of the floor) , then the «support reaction» forces are the normal forces at each of the legs of the chair. That is one example of a support reaction but there are many examples of supports that have different reaction characteristics, and in mechanics, it is very important to understand how to model these reaction forces.

The forces of viscous (internal) friction $F_в$ are friction forces that arise when the body moves in a viscous (liquid or gaseous) medium. With a small value of the velocity v of motion of the body relative to the medium: $F_в = -\eta v$, where η is the coefficient of viscous (internal) friction.

The force of viscous friction of rest is zero: $F_{вп} = 0$.

The forces of dry friction F_c are friction forces arising from direct contact of solids.

The forces of friction of rest $F_п$ are the dry friction forces that arise in the absence of the relative motion of interacting bodies.

The friction force of sliding $F_{ск}$ is the force of dry friction that occurs when the interacting bodies move relative.

The Amonton-Coulomb law is an empirical law that describes the properties of dry friction forces:

1) the modulus of the dry friction force of rest can take values from zero to some maximum value: $0 \le F_п \le F_{\max}$;

2) the modulus of the dry friction force of sliding is equal to the maximum value of the modulus of the dry friction force of rest: $F_{ск} = F_{\max}$;

3) the modulus of the dry friction force of sliding is proportional to the modulus of the normal pressure force: $F_{ск} = \mu N$, where μ is the coefficient of dry friction force, independent of the force of normal pressure, but only dependent of the substance and state of the surfaces of rubbing bodies;

4) the dry friction force of sliding is directed opposite to the direction of speed of relative motion of bodies $v_{отн}$: $F_{ск} \uparrow\downarrow v_{отн}$.

A force field is a region of space where forces of a given nature act. In general case they depend on time, on the coordinate and on velocity of the motion of the material point: $F(t, r, v)$.

2. Basic types of kinematic and dynamic problems

a) Classification of problems of kinematics

The main problem of kinematics is determination of the kinematic characteristics of bodies moving relative to a given system of reference.

Most kinematics problems can be conditionally attributed to the following types of tasks or their combinations:

1) kinematics of a material point,
2) the principle of superposition of motions,
3) kinematic coupling equations,
4) kinematics of the simplest mechanical systems.

As a rule, one of the types of problems is basic problem, while others are subordinate to the condition of the problem value.

b) Classification of problems of dynamics

The direct problem of dynamics is the determination of the law of motion of a body or system of bodies if the forces acting on these bodies are known.

The inverse problem of the dynamics of a material point consists in finding the forces acting on the body or system of bodies, if the laws of motion of these bodies are known.

Most problems contain elements of both direct and inverse dynamic problems. As a rule, one of these tasks has a basic, the other - subordinate to the condition of the task value.

3. General scheme for solving kinematic and dynamic problems

a) *General scheme for solving kinematic problems*

I. Identify with models of material objects and phenomena.

1. Draw a drawing on which to depict the bodies in question.
2. Select the reference system and depict in its drawing its coordinate system (for convenience reasons).
3. Depict and designate the kinematic characteristics of bodies.
4. Select models of bodies and their movements (if this is not done in the condition of the problem).

I. Write the complete system of equations for the unknown quantities.

1. Record in the projections on the coordinate axes:

a) the laws of motion,
b) the laws of velocity variation,
c) laws of acceleration variation.

2. Write down the initial conditions.
3. Write down the equations of kinematic constraints.
4. Use the results of previously solved problems and the special conditions of the problem (for example, the given relations between the characteristics of the system).

III. Get the desired result in analytical and numerical forms.1. Solve the system of obtained equations.

2. Analyze the solution (check the dimension and extra roots, consider the characteristic cases, establish the region of applicability).
3. Obtain a numerical result.

Notes. In the case of solving problems on the kinematics of the material point in points. I.3 - II.2 we are talking about the kinematic characteristics of a material point, and the point II.3 should be omitted.

In the case of solving problems on the kinematics of the simplest mechanical systems, in points I.3 - II.2 we are talking about the kinematic characteristics of the bodies of the system under consideration.

Points II.1 - II.3 (including II.2.a - II.2.b) can be implemented in a sequence depending on the type of task.

b) General scheme for solving dynamic problems with the help of Newton's laws

I. Identify with models of material objects and phenomena.

1. Draw a drawing on which to depict the considered bodies.
2. Select the reference system and depict in the drawing its coordinate system (for reasons of convenience).
3. Depict and designate all forces and necessary kinematic characteristics of the system.
4. Select models of bodies and their movements (if this is not done in the condition of the problem).

II. Write the complete system of equations for the unknown quantities.

1. Write the equations of motion in projections on the coordinate axes for all bodies of the system.

2. Use Newton's third law, if this was not done previously in the point 3.

3. Use laws that describe the individual properties of forces:

a) the law of universal gravitation,

b) Hooke's law,

c) the Amonton-Coulomb law, etc.

4. Write down the equations of kinematic constraints.

5. Use the results of previously solved problems and the special conditions of the problem.

III. Get the desired result in analytical and numerical forms.

1. Solve the system of equations.

2. Analyze the solution (check the dimension and extra roots, consider the limiting and special cases, establish the region of applicability).

3. Obtain a numerical result.

Notes. In the case of solving problems on the dynamics of a material point in points I.3 - II.1 we are talking about the characteristics of a material point, and paragraph II.2 should be omitted. In the case of solving problems on the dynamics of the simplest mechanical systems, in points I.3 - II.2 we are talking about the characteristics and equations of motion of bodies and forces acting between the bodies of the system under consideration. Items II.1 - II.4 (including II.3.a - II.3.c) can be performed in a sequence depending on the task to be accomplished [5].

Conclusion. In classical mechanics, acquiring a conceptual understanding of problem has proven to be one of the most difficult challenges faced by students and master students. Teaching and learning methods used in classical mechanics and mechanical analysis have traditionally been very theoretical, and reference material mirrors this approach. This work illustrates teaching method that reacts to the need for increased conceptual understanding include the use of the theoretical material, knowledge of basic types of problems and models or schemes for problem solving. Experience shows that this practical approach, used by many teachers of mechanics has been shown to be a promising way forward.

References

1 Дронг В.И. Курс теоретической механики / В.И. Дронг, В.В. Дубинин, М.М. Ильин, К.С. Колесников и др.; под ред. К.С. Колесникова. — М.: Изд-во МГТУ им. Н.Э. Баумана, 2005. — 736 с.

2 Трухан Н.М. Кинематика. Методические указания по решению задач по курсу: «Теоретическая механика» / Н.М. Трухан. — М.: МФТИ, 1991. — 32 с.

3 Акимов В.А. Теоретическая механика. Динамика. Практикум. Ч. 2 / В.А. Акимов. — Минск: Новое знание; — М.: ЦУПЛ, 2010. — 863 с.

4 Митюшов Е.А. Теоретическая механика: Статика. Кинематика. Динамика / Е.А. Митюшов, С.А. Берестова. — М.;-Ижевск: Ин-т компьютерных исследований, 2005. — 176 с.

5 Русаков В.С. Механика. Методика решения задач / В.С. Русаков, А.И. Слепков, Е.А. Никанорова, Н.И. Чистякова. — М.: Физический факультет МГУ, 2010. — 368 с.

Г.А. Есенбаева, Д.Н. Есбаева, К.С. Кутимов, Д.А.-Х. Нургали, Т.Х. Макажанова

Кинематика мен динамиканың механикалық тапсырмаларын шешу тәсілдерінің бірі туралы

Мақалада механикаға нақты ғылымдарда қолданылатын мәселелерді шешудің бір әдісін қолдану туралы айтылған. Бұл әдіс құрылымының негізгі компоненттері анықталған және механиканың мәселелерін шешудің негізгі кезеңдері сипатталған. Дәл ғылымдардағы мәселелерді шешуге деген көзқарас, жалпы айтқанда, классикалық болып табылады, бірақ осы әдісті белгілі бір пәндік салаға қолдану, осы мақалада көрсетілгендей, практикалық қолданудың ерекшеліктерін тудырады. Зерттелетін мәселелерді шешуге деген тәсілі кинематика мен динамикадағы механикалық мәселелерге арналған. Басты назар осы әдістердің негізгі сатыларына бағытталған. Кинематика және динамика бойынша теориялық материал қысқа көлемде мәселелерді шешу үшін қажетті арнайы таңдап алынған және тиімді түрде тұжырымдалған. Кинематика және динамика мәселелерін жіктеу кезінде олардың негізгі түрлері берілген. Кинематика және динамика мәселелерін шешуге арналған құрылымдар жалпы түрде жасалды, бірақ ескертулер нақты жағдайларға арналған ұсынылған шешім құрылымдарын пайдалану туралы түсінік берді. Мақалада механиктер, физиктерге және техникалық мамандықтарда оқитын студенттерге арналған.

Кілт сөздер: кинематика, динамика, күштер, қозғалыс теңдеулері, кинематика мен динамиканың негізгі түрлері, кинематика мен динамика мәселелерін шешу схемасы.

Г.А. Есенбаева, Д.Н. Есбаева, К.С. Кутимов, Д.А.-Х. Нургали, Т.Х. Макажанова

Об одном подходе к решению механических задач кинематики и динамики

Статья посвящена вопросу приложения к механике одного из методов решения задач, применяемых в точных науках. В статье представлена структура данного метода, выделены и охарактеризованы ее основные компоненты, отмечены моменты, на которые следует обратить внимание, описаны основные этапы решения задач механики. Рассматриваемый подход к решению задач в точных науках, вообще говоря, является классическим, однако применение этого метода к конкретной предметной области влечет за собой специфические особенности практического приложения, которые отмечены в данной статье. Исследуемый подход решения задач продемонстрирован для механических задач по кинематике и динамике. Основное внимание акцентируется на ключевых этапах данного метода. Теоретический материал по кинематике и динамике специально подобран и рационально сформулирован в необходимом для решения задач кратком объеме. В классификации задач кинематики и динамики даны их основные типы. Схемы решения задач кинематики и динамики сформулированы в наиболее общей форме, но в замечаниях даются пояснения использования представленных схем решения для частных случаев. Статья ориентирована главным образом на студентов — механиков, физиков и обучающихся на технических специальностях.

Ключевые слова: кинематика, динамика, силы, уравнения движения, основные типы задач кинематики и динамики, схемы решения задач кинематики и динамики.

References

1 Drong, V.I., Dubinin, V.V., Ilyin, M.M., Kolesnikov, K.S., & et al. (2005). *Kurs teoreticheskoi mekhaniki [Course of theoretical mechanics]*. K.S. Kolesnikov (Ed.). Moscow: Izdatelstvo MHTU imeni N.E. Baumana [in Russian].

2 Trukhan, N.M. (1991). *Kinematika. Metodicheskie ukazaniia po resheniiu zadach po kursu. Teoreticheskaia mekhanika [Kinematics. Methodical instructions for solving problems on the course. Theoretical Mechanics]*. Moscow: MPhTI [in Russian].

3 Akimov, V.A. (2010). *Teoreticheskaia mekhanika. Dinamika. Praktikum [Theoretical mechanics. Dynamics. Practical work]*. (Part 2). Minsk: MFTI [in Russian].

4 Mityushov, E.A., & Berestova, S.A. (2005). *Teoreticheskaia mekhanika: Statika. Kinematika. Dinamika [Theoretical mechanics: Statics. Kinematics. Dynamics]*. Moscow-Izhevsk: Institut kompiuternykh issledovanii [in Russian].

5 Rusakov, V.S., Slepkov, A.I., Nikanorova, E.A., & Chistyakova, N.I. (2010). *Mekhanika. Metodika resheniia zadach [Mechanics. Methods of solving problems]*. Moscow: Fizicheskii fakultet MHU [in Russian].