UDC 530.1; 51-37

ITERATIVE CALCULATION OF NONLINEAR INTERACTION AND MOTIONS OF PLANETS IN SOLAR SYSTEM

[1]T.A.Zhakatayev, [2]A.T. Zhakatayev

[1]Karaganda State University named after E.A.Buketov, Universitetskaia Str. 28, Karaganda, 100026, Kazakhstan, Toksanzh@yandex.ru, [2]Texas A&M University

*The system of the equations which models nonlinear gravitational interaction of several planets in solar system is made. The system of the equations is solved numerically on package Maple 10. The effect of nonlinear change of parameters as a result of gravitational interaction is found out. Nonlinear gravitational interaction leads to that orbital speed changes. Accordingly, cycle times of planets change depending on quantity of planets and a configuration of their orbits in system.***

Keywords: solar system, gravitational interaction, cycle times, orbital speeds, orbital periods.

Let's consider classical solution when movement on a circular orbit is described on the basis of balance of forces provided. Thus centripetal force is defined through the acceleration of gravity toward the center attraction

$$\frac{m_1 v^2}{r} = \gamma\, m_1 m_2 \big/ r^2, \tag{1}$$

here γ=6,672·10^{-11} H·m^2/кg^2. Here is not considered nonlinear effects of interaction at orbital movement. Methods of classical mechanics allow analytical solution only for a problem of gravitational interaction of two bodies [1-5]. Numerical methods have great prospects in connection with rapid development of computers, when finding of unknown function is reduced to iterative process. Therefore in the present work we have considered the general motion of: 1) two planets (the Earth-the Mars; the Earth- the Venus; the Earth-the Jupiter); 2) three and more planets (the Earth with the Moon, the Venus, the Jupiter and the Mars) in the central gravitational field of the Sun. Thus this problem is viewed as the nonlinear gravitational interaction of three and more bodies. As the initial conditions we have accepted an arrangement of all planets at initial moment on the axis X, and vectors of speed of these planets have been accepted to be directed on the axis Y. This problem is only a model for demonstration of application of an iterative method. Basically the given technique and the given scheme are easily generalized to the problems of interaction of any quantity of bodies (n=4,5,6 ….). Hence we state the decision of this problem only as demonstration of the method. The solution of similar problems is actual in space technology. For example it is possible to calculate precisely nonlinear interaction and motion of the artificial satellite both around the Earth, and around other planets.

The general picture of movement of planets in solar system is shown in figure 1 [6].

The equations of motion for considered system (in vector notation) have the form

$$m_2 \frac{d^2\vec{r}_2}{dt^2} = -\gamma \frac{m_1 m_2}{\vec{r}_2^{\,2}}\bar{r}_2 + \gamma \frac{m_2 m_3}{\vec{r}_{23}^{\,2}}\bar{r}_{23}, \tag{2}$$

$$m_3 \frac{d^2\vec{r}_3}{dt^2} = -\gamma \frac{m_1 m_3}{\vec{r}_3^{\,2}}\bar{r}_3 - \gamma \frac{m_2 m_3}{\vec{r}_{23}^{\,2}}\bar{r}_{23}, \tag{3}$$

where m_1 - mass of the Sun, m_2 - mass of the Earth, m_3 - mass of the third planet (Mars, Venus or Jupiter).

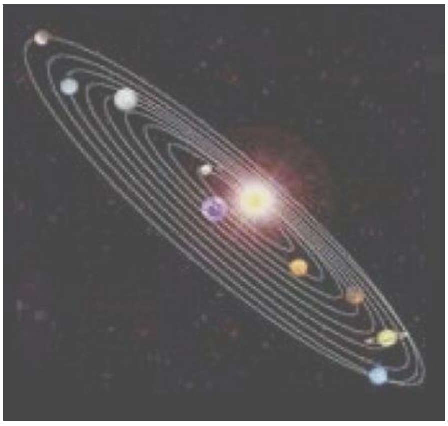

Fig. 1. General view of solar system [6]

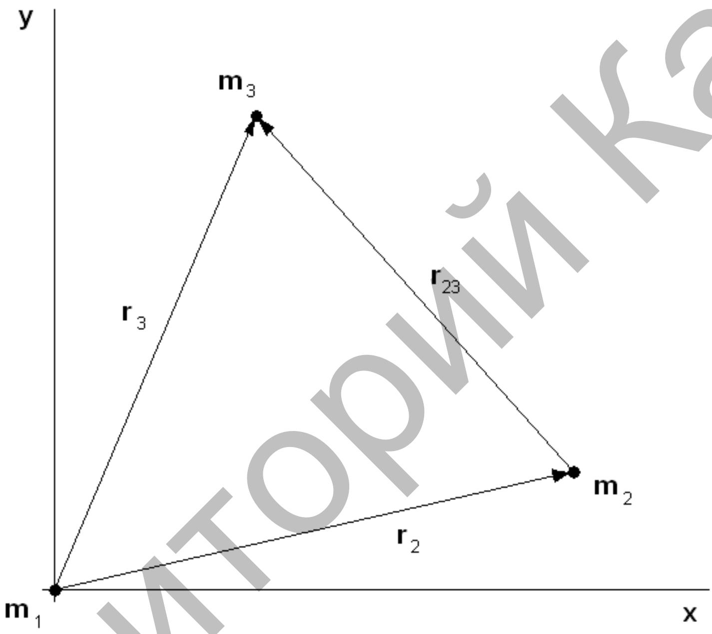

Fig. 2. System of coordinates for a problem of three bodies, at the center of the system is the Sun. Problem № 1

Using the vector identities

$$\vec{r}_2 = x_2\bar{i} + y_2\bar{j},\ \vec{r}_3 = x_3\bar{i} + y_3\bar{j},\ \vec{r}_{23} = (x_3 - x_2)\bar{i} + (y_3 - y_2)\bar{j} \qquad (4)$$

and from (2), (3) we shall receive the following scalar equations

$$\frac{d^2x_2}{dt^2} = -\gamma\frac{m_1}{(x_2^2 + y_2^2)^{3/2}}\cdot x_2 + \gamma\frac{m_3}{\left[(x_3 - x_2)^2 + (y_3 - y_2)^2\right]^{3/2}}\cdot(x_3 - x_2), \qquad (5)$$

$$\frac{d^2y_2}{dt^2} = -\gamma\frac{m_1}{(x_2^2 + y_2^2)^{3/2}}\cdot y_2 + \gamma\frac{m_3}{\left[(x_3 - x_2)^2 + (y_3 - y_2)^2\right]^{3/2}}\cdot(y_3 - y_2), \qquad (6)$$

$$\frac{d^2x_3}{dt^2} = -\gamma\frac{m_1}{(x_3^2 + y_3^2)^{3/2}}\cdot x_3 - \gamma\frac{m_2}{\left[(x_3 - x_2)^2 + (y_3 - y_2)^2\right]^{3/2}}\cdot(x_3 - x_2), \qquad (7)$$

$$\frac{d^2y_3}{dt^2} = -\gamma\frac{m_1}{(x_3^2+y_3^2)^{3/2}}\cdot y_3 - \gamma\frac{m_2}{\left[(x_3-x_2)^2+(y_3-y_2)^2\right]^{3/2}}\cdot(y_3-y_2). \qquad (8)$$

We approximate derivatives with finite differences [9 - 10]

$$\frac{d^2x_2}{dt^2} \approx \frac{x_{2,i-1}-2x_{2,i}+x_{2,i+1}}{h_t^2}, \qquad (9)$$

$$x_{2,i} = x_{2,0} + i\cdot h_t, \qquad (10)$$

where h_t - a step size in time, s, with i=0,1,2, … - an index of a step. Expressions for derivatives y_2, x_3, y_3 have a similar appearance. On the basis of (9) the differential equations (5) - (8) are easily led to the following iterative equations

$$x_{2,i}^{(k)} = \varphi_1(x_{2,i-1}^{(k-1)}, x_{2,i}^{(k-1)}, x_{2,i+1}^{(k-1)}),$$

$$y_{2,i}^{(k)} = \varphi_2(y_{2,i-1}^{(k-1)}, y_{2,i}^{(k-1)}, y_{2,i+1}^{(k-1)}),$$

$$x_{3,i}^{(k)} = \varphi_3(x_{3,i-1}^{(k-1)}, x_{3,i}^{(k-1)}, x_{3,i+1}^{(k-1)}),$$

$$y_{3,i}^{(k)} = \varphi_4(y_{3,i-1}^{(k-1)}, y_{3,i}^{(k-1)}, y_{3,i+1}^{(k-1)}). \qquad (11)$$

where

$$\varphi_1(x_{2,i-1}^{(k-1)}, x_{2,i}^{(k-1)}, x_{2,i+1}^{(k-1)}) = \frac{1}{2}x_{2,i-1} + \frac{1}{2}x_{2,i+1} + \frac{h_t^2\gamma m_1}{2}\cdot\frac{x_{2,i}}{(x_{2,i}^2+y_{2,i}^2)^{3/2}} -$$

$$\frac{h_t^2\gamma m_3}{2}\cdot\frac{(x_{3,i}-x_{2,i})}{\left[(x_{3,i}-x_{2,i})^2+(y_{3,i}-y_{2,i})^2\right]^{3/2}},$$

$$\varphi_2(y_{2,i-1}^{(k-1)}, y_{2,i}^{(k-1)}, y_{2,i+1}^{(k-1)}) = \frac{1}{2}y_{2,i-1} + \frac{1}{2}y_{2,i+1} + \frac{h_t^2\gamma m_1}{2}\cdot\frac{y_{2,i}}{(x_{2,i}^2+y_{2,i}^2)^{3/2}} -$$

$$\frac{h_t^2\gamma m_3}{2}\cdot\frac{(y_{3,i}-y_{2,i})}{\left[(x_{3,i}-x_{2,i})^2+(y_{3,i}-y_{2,i})^2\right]^{3/2}}, \qquad (12)$$

$$\varphi_3(x_{3,i-1}^{(k-1)}, x_{3,i}^{(k-1)}, x_{3,i+1}^{(k-1)}) = \frac{1}{2}x_{3,i-1} + \frac{1}{2}x_{3,i+1} + \frac{h_t^2\gamma m_1}{2}\cdot\frac{x_{3,i}}{(x_{3,i}^2+y_{3,i}^2)^{3/2}} +$$

$$\frac{h_t^2\gamma m_2}{2}\cdot\frac{(x_{3,i}-x_{2,i})}{\left[(x_{3,i}-x_{2,i})^2+(y_{3,i}-y_{2,i})^2\right]^{3/2}}, \tag{14}$$

$$\varphi_4(y_{3,i-1}^{(k-1)}, y_{3,i}^{(k-1)}, y_{3,i+1}^{(k-1)}) = \frac{1}{2}y_{3,i-1} + \frac{1}{2}y_{3,i+1} + \frac{h_t^2\gamma m_1}{2}\cdot\frac{y_{3,i}}{(x_{3,i}^2+y_{3,i}^2)^{3/2}} -$$

$$\frac{h_t^2\gamma m_2}{2}\cdot\frac{(y_{3,i}-y_{2,i})}{\left[(x_{3,i}-x_{2,i})^2+(y_{3,i}-y_{2,i})^2\right]^{3/2}}. \tag{15}$$

In this way we have led the decision of an initial problem to iterative process x=φ(x).

The system (5-8) is solved on the Maple 10 package. In this package the system of the second order differential equations by means of new additional variables $x' = z; y' = v$ is led to the expanded system of the ordinary differential equations of the first order. Further the Runge-Kutta scheme [9, 10] is used.

Problem №1. Calculations were done as following. At the beginning the period of rotation of the Earth around the Sun in the absence of influence of all the other planets is calculated. The system is considered only to be: 1) Sun+Earth. After that calculation in the system of three bodies is made: 2) Sun+Earth+Venus, 3) Sun+Earth+Mars, 4) Sun+Earth+Jupiter. The results of calculation in systems 2-4 are compared with the result in system 1. In system 2 period of one full turn of the Earth around the Sun (in comparison with 1) increases on 979 second.

In system 3 period of one full turn of the Earth (in comparison with 1) decreases to 87 second, while in system 4 time decreases to 7920 second. This way the following nonlinear effect is found out. At the indicated initial conditions internal planets “slow down” motion of the Earth, while external planets “accelerate”. It is possible to explain this effect on the basis of the gravitational law (1). Whence follows, that

$$\omega^2 r^3 = \text{const}. \tag{16}$$

From here it is clear that when radius decreases, angular frequency increases. When frequency increases the period accordingly decreases. And on the contrary, at increase of circular radius the frequency decreases, as a result the period increases. At the indicated initial conditions, internal planets actually accelerate the Earth. However, when Earth gains speed, it gets to elliptical orbit with higher semi-major axis, which has longer period of rotation around the Sun. Therefore it seems as internal planets “slow down” the Earth. Vice versa true for external planets.

Calculations are done using the following data in table 1.

Table 1

Object	Mass, kg	Initial radius of an orbit, m.	Initial orbital speed, km/s
The Earth	$5{,}9736\times10^{24}$	$149{,}6\times10^{9}$	29,783
The Venus	$4{,}8685\times10^{24}$	$108{,}209\times10^{9}$	35,02
The Mars	$6{,}4185\times10^{23}$	$227{,}94\times10^{9}$	24,077
The Floodlight	$1{,}8986\times10^{27}$	$778{,}55\times10^{9}$	13,07
The Sun	$1{,}99\times10^{30}$		
The Moon	$7{,}3477\times10^{22}$	$384{,}4\times10^{6}$ relative to the Earth	1,022 relative to the Earth

Figure 3 shows the system of coordinates for a problem 2.

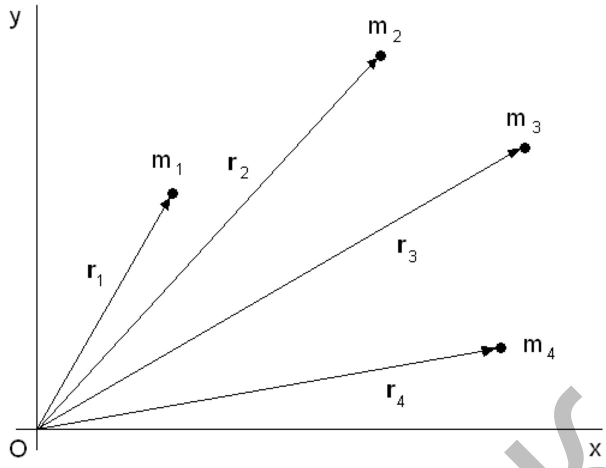

Fig. 3. System of coordinates for a problem № 2

$$\vec{r}_k = x_k \bar{i} + y_k \bar{j}, k = 1,2,3... \qquad (17)$$

In this case the Sun is described by a vector $\vec{r}_1$, m1 - its mass. During the initial moment of time $\vec{r}_1(x_1, y_1)$ =0. After some time coordinates of the Sun receive some value distinct from zero. Figure 4 shows precession movement of the center of the Suns for a problem №2. Precession is described by change of radius-vector $\vec{r}_1$ in time t.

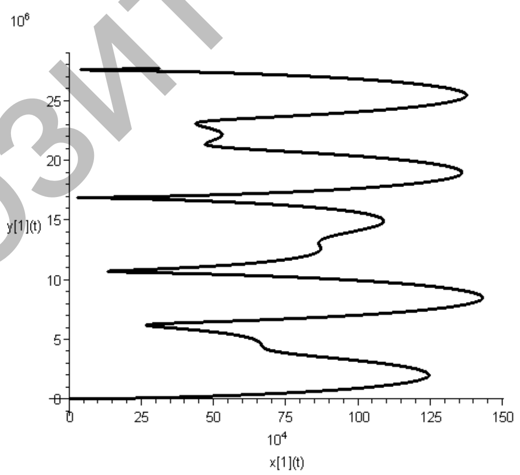

Fig. 4. Coordinates of the center of the Sun in precession movement, the system: the Sun-Earth-Moon-Venus. The problem № 2

In figure 5 are shown trajectories of the Earth (1) and the Moon (2) relative to coordinate system №2 (figure 3). From here it is visible, that the calculating model correctly describes circular-oscillatory motion of the Moon relative to the Earth.

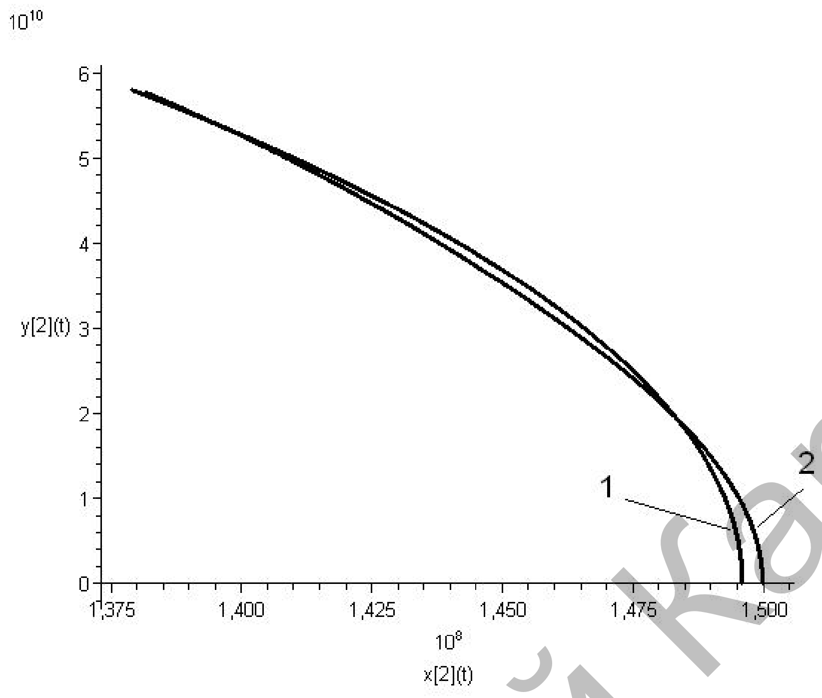

Fig. 5. Trajectories of motion of the Earth (1) and the Moon (2) relative the coordinate system №2. The problem № 2

The differential equations of motion for a problem №2 have the following form

$$\begin{aligned}
m_1\frac{d^2\vec r_1}{dt^2} &= \frac{\gamma m_1 m_2}{\vec r_{12}^{\,2}}\bar r_{12} + \frac{\gamma m_1 m_3}{\vec r_{13}^{\,2}}\bar r_{13} + \frac{\gamma m_1 m_4}{\vec r_{14}^{\,2}}\bar r_{14},\\
m_2\frac{d^2\vec r_2}{dt^2} &= \frac{\gamma m_2 m_1}{\vec r_{21}^{\,2}}\bar r_{21} + \frac{\gamma m_2 m_3}{\vec r_{23}^{\,2}}\bar r_{23} + \frac{\gamma m_2 m_4}{\vec r_{24}^{\,2}}\bar r_{24},\\
m_3\frac{d^2\vec r_3}{dt^2} &= \frac{\gamma m_3 m_1}{\vec r_{31}^{\,2}}\bar r_{31} + \frac{\gamma m_3 m_2}{\vec r_{32}^{\,2}}\bar r_{32} + \frac{\gamma m_3 m_4}{\vec r_{34}^{\,2}}\bar r_{34},\\
m_4\frac{d^2\vec r_4}{dt^2} &= \frac{\gamma m_4 m_1}{\vec r_{41}^{\,2}}\bar r_{41} + \frac{\gamma m_4 m_2}{\vec r_{42}^{\,2}}\bar r_{42} + \frac{\gamma m_4 m_3}{\vec r_{43}^{\,2}}\bar r_{43}.
\end{aligned} \tag{18}$$

The system (18) can be presented in more compact form

$$\frac{d^2 x_i}{dt^2} = \sum_{j\neq i}\gamma\frac{m_j(x_j - x_i)}{\left((x_j - x_i)^2 + (y_j - y_i)^2\right)^{3/2}},$$

$$\frac{d^2 y_i}{dt^2} = \sum_{j\neq i}\gamma\frac{m_j(y_j - y_i)}{\left((x_j - x_i)^2 + (y_j - y_i)^2\right)^{3/2}}.$$

The problem №2 for system (19) (as a problem №1) is solved by using the Maple 10 package.

Calculations were made as the following. At the beginning the period of rotation of the Earth (together with the Moon) around the Sun is calculated at the absence of influence of all the other planets. That is the system is: 1) Sun+Earth+Moon. After that calculations were done in system of three and more bodies: 2) Sun+Earth+Moon+Mars, 3) Sun+Earth+Moon+Venus, 4)

Sun+Earth+Moon+ Venus+Jupiter. The results of computations in systems 2-4 are compared with that of system 1. In system 2 period of one full turn of the Earth around the Sun (in comparison with 1) decreases to 62 second. In system 3 period of one full turn of the Earth (in comparison with 1) increases to 122 second. In system 4 it decreases to 368 second. Hence above mentioned nonlinear effect appears again. Under the given initial conditions, internal planets "slow down" motion of the Earth, and external planets "accelerate". Presence of the Moon reduces this nonlinear effect.

We have received expression for a step size of iteration h_t (an estimation from above), but it is too bulky to present it in this article. Numerical value for a step size iteration for the case Sun+Earth+Mars has appeared to be equal to $2{,}33 \cdot 10^{14}$ second. It gives big range for variation of a step size.

Conclusions

1. The system of the differential equations which models nonlinear gravitational interaction of several planets in the solar system is solved numerically on the Maple 10 package. The effect of nonlinear change of parameters as a result of gravitational interaction is found out.

2. Nonlinear gravitational interaction leads to changes of orbital speeds of the planets. Accordingly, orbital periods of planets change depending on quantity of planets and a configuration of their orbits in system**.**

References:

1. Olchovskij I. I. Course of theoretical mechanics for physiker.-M.: Publishing house of the Moscow University, 1978. - 3 - publication. - 574 p.
2. Lojtsjanskij L.G., Lurje A. I. Course of theoretical mechanics.-M.: The Science, 1983. **Vol.2**.- 640 p.
3. Multanovskij V. V. Course of Theoretical physics. Classical mechanics.-M.: Education, 1988.-304 p.
4. Berezkin E. N. Course theoretical mechanics.-M.: Publishing house of the Moscow University, 1974.-2 publication. - 646 p.
5. Schaub H., Junkins J. Analytical mechanics of Space Systems.-Virginia, 2003. AIAA education series.-716p.
6. http://www.nightskynation.com/pics/planets.jpg
7. Samarskij A. A. Leading in the theory difference scheme.-M.: Science, 1971.-552 p.
8. Krylov V.I., Bobkov V.V., Monasturnui P. I. Computing methods. - M.: The Science, 1976. - **Vol. 1**.-303 p.
9. Berezin I.S., Zhidkov N. P. Methods of calculations. .- M.: St. publishing house of the physical and mathematical literature, 1959.-**Vol. 2**.-620 p.
10. Krylov V.I., Bobkov V.V, Monasturnui P. I. Computing methods. - M.: The Science, 1977. - **Vol. 2**. - 400 p.