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On teaching of bases of real variable functions theory

The main aim of the work is one of the mathematics disciplines teaching methodology improving, namely the real variable functions theory. The article outlines the theoretical material - basic concepts, statements in their logical sequence. For practical lesson learning task is chosen, which implies the need for the student to identify a particular something in common, as discussed in the lecture. The teacher guides the work of students c through questions to encourage them to self-reasoning and active search for the right solutions. In this article we present the test questions and tasks to determine the rate of formation of the competence of the student. Thus, we believe that the study is transformed from a traditional discipline of study in cooperation and conscious acquisition of knowledge. The paper presents one of the methods of presentation of theoretical material - basic concepts, statements in their logical sequence. For the practical sessions as the theme chosen this learning task, which involves the need for the student to "touch" material, in particular to identify common, as discussed in the lecture. In the practical classes the teacher directs the work of students c through questions to encourage them to self-reasoning and active search for the right solutions. In this regard, we give control questions and tasks to determine the rate of formation of the competence of the student. Thus, we believe that the study is transformed from a traditional discipline of study in cooperation and conscious acquisition of knowledge.

Keywords: problem educating, constructive educating, competence, interactive form of educating, continuity, measurement of sets.

Elementary knowledge of real variable functions theory is the necessary part of mathematical culture of future teacher. Discipline "Theory of real variable functions" refers to the cycle of fundamental mathematical disciplines. The study of this discipline is based on the "Mathematician" knowledge the students in frames of high school, and also such mathematical disciplines as: "Analytical geometry", "Linear algebra", "Calculus". Free possession is assumed also the basic concepts of Calculus, such as a limit, derivative, integrals and series. However, knowledge of these concepts in the volume of course of Calculus is not always enough for the modern applied and theoretical tasks solution.

Therefore there is a necessity of knowledge expansion for the following: students' structural thinking development, mathematical knowledge forming for a successful capture by professional skills at necessary scientific level. In addition, substantive provision of discipline "Theory of real variable functions" is foundation of mathematical formation of the applied mathematician education, important for the successful study of general mathematical and special disciplines.

The content of discipline supposes the study of the following questions: «Capacity of set», «Countable and countless sets», «Structure of the closed and open sets on a numerical line», «Concept of metrical space», «Complete metrical spaces», «Measure of Lebesgue», «Sets and functions», «Measurable on Lebesgue», «Lebesgue integral», «Fourier series».

For better understanding of course of real variable functions theory , for students' creative independence development, stressful analysis of basic concepts, methods and theorems of mathematical analysis may serve the tasks of studying and scientific research character, because they have the direct exit on serious mathematical research.

The solution of these tasks requires the independent mathematical reasoning, acquaintance and working of scientific methodological literature, ability to process scientific information, make independent conclusions.

The theory of real variable functions is one of the most important studied subjects at physics and mathematics faculties in pedagogical universities. A teacher constantly meets concepts of a set, real number, function, limit, continuity, measurement of sets which form the maintenance of this subject during work. It is impossible to conduct teaching any school course in mathematics at a relevant scientific level, without knowing the bases of the real variable functions theory, the ideas of which cover the whole areas of mathematics.

Psychologists assert that the source of the creative thinking and its beginning is the difficult situation, both theoretical and practical, that requires the search of solution and, certainly, research. The educating problem is a method, during that the serve of new material takes place through creation of problem situation that is for a student intellectual difficulty.

Traditionally, the lecturer delivers already prepared material, thus, giving the students the supposed answers. Knowledge got in the process of lecture listening are percept almost always in the appropriate manner. But that's definitely the mechanical knowledge acquisition. Therefore the question: is there student's initiative in such knowledge receipt? Practically both methods (problem and traditional) educating are used by teachers together, they complement each other. However the teacher‘s role here is not diminished, it is increased instead.

Pedagogy gives an opportunity for enormous amount of methods and problem situation introduction variants in the process of educating. They assist the variety in educating, i.e. possibilities to choose the variant of material giving out.

In this article we try to review another method of teaching to basic concepts of the point set theory. Meantime, we want to examine various point sets, the sets and elements of which points either a numerical straight line or a point of any n-dimensional Euclidean space.

As one-to-one correspondence between set of real numbers and all point sets of numerical straight line, studying of linear dot sets that are point sets of a straight line determined is identical to studying of the sets consisting of real numbers. While teaching this course, it is essential to introduce the definitions of the simplest and most common point sets, segment, interval, semi-interval.

It is also necessary that students clearly understand due to one-to-one correspondence between the set of all real numbers and the set of all points of the real line the segment definition, the interval and interspaces for them are identical to the definitions of numerical sets. Then the definition of contracting sequence of segments and intervals are introduced [1].

The concepts of a segment and an interval extend in multidimensional spaces. Thus, under the segment of two-dimensional space, i.e. in the plane we mean the set of all points (*x, y*) of plane, where each of the coordinate values form a line segment, for example $a \le x \le b$ and $c \le y \le d$. So, the two-dimensional segment is a set of all points of the plane which are inside and on the some parts of rectangle. We understand a set of all points (x, y, z) for that we have $a \le x \le b, \quad c \le y \le d, \quad k \le z \le l$, as an interval of three-dimensional space that is the three-dimensional interval is a set of all points, containing in some parallelepiped. Therefore, it is demandable to introduce definition of an *n*-dimensional segment, *n*-dimensional interval and to show under what conditions sequence of *n*-dimensional segments will be constructed.

1-Theorem [2]. If the sequence of segments is being constructed, there is only one point belonging to all segments.

The theorem can be proved both for a case of linear segments, and for the general case. It should be noted that the theorem refers to a sequence of segments. That is a required condition. So, for example, $(0 < x \le 1]$, $(0 < x \le 1/2]$,..., $(0 < x \le 1/n]$,... sequence of linear semi-intervals, by construction, but has no common point.

Furthermore, the basic concepts of the point sets theory and its definition are introduced: bounded set, neighborhood, the limit point of the set, isolated point of the set and the theorem, which gives sufficient conditions for the existence of a limit point of the set.

2-Theorem (Bolzano-Weierstrass) [2]. Every bounded infinite set has, at least, one limit point.

It should be noted that here the condition of limitation of a set is essential, without this condition theorem is not true. Thus, the infinite unbounded set $Z = \{..., -2, -1, 0, 1, 2, ...\}$ does not have any limit point. At the same time, there are unlimited infinite sets, limit points. The set of all points, for example of a straight line will be such. Consequently, limitation of a set is sufficient, but not necessary condition for the existence of a limit set point.

Based on Bolzano-Weierstrass's theorem the assertion of the statement concerning the infinite sequences is implied, so it is necessary to introduce the definition of point's sequence of *n*-dimensional space, a convergent sequence and subsequence.

Consequence. From every bounded sequence a convergent subsequence can be selected.

Further, the definitions of still number of important concepts of the point set theory are introduced: derivative set, closed set, dense set in itself, perfect set, internal point of set, open set, closure of set. It should be noted that the set can be at the same time both closed, and opened, and also it can be at the same time both not closed, and not opened, i.e. concepts of the closed and open set are connected among themselves.

When the closed sets theorem is considered and introduced, the nature of the derived set of any set of points is revealed.

3-Theorem [2]. Arbitrary set *G* of any closed set *D* is the closed set.

The following two theorems show at which cases the operations of addition and intersection of sets will not remove them from the class of closed sets. And sets about which there is a speech in these theorems, consist of points of n-dimensional Euclidean space, particularly, can be linear.

4-Theorem [2]. Sum of a finite set of closed sets is the closed set.

It is necessary to pay attention that the sum of an infinite set of closed sets can not be the closed set.

For example [3], set $[1;\,1/2],[1/2;\,1/3],[1/3;\,1/4],...[1/n;\,1/(n+1)],..$ is closed like segments, but their sum is semi-interval [1; 0) is not closed set, because point 0 is not contained in any of the components, and therefore sum equals:

$$[1;\,0)=[1;\,1/2]+[1/2;\,1/3]+[1/3;\,1/4]+...+[1/n;\,1/(n+1)]+...$$

5-Theorem [2]. Intersection of any sets of closed sets is the closed set.

Note that the intersection of closed sets may be empty. For instance, two segments may not have any common points. This does not contradict the proved theorem, since the empty set is closed.

6-Theorem (*Borel*) *[2].* From any infinite system *G* of intervals *t* covering the limited closed set *F* it is possible to allocate the finite system *D* of intervals t which also covers a set *F*. It should be noted that theorem conditions – isolation and limitation of this set – are essential. Suppose, for example, we have closed, but not limited set $N=\{1,2,...,n,...\}$. Each point $n\in N$ will cover t_n such an interval that will not contain other points *N*.

It is obvious that it is possible as all points of *N* are isolated, then we will receive infinite system of intervals *N* covering all set. Obviously, it is possible, as all points of *N* are isolated, and then we obtain an infinite system of intervals $G=\{t_1,t_2,...,t_n,...\}$, covering all sets of *N*. It is clear, that finite part *G* can't cover all infinite set *N*, as in each interval t_n only one point of a set *N* is contained. All this can be repeated for limited, but not closed set $E=\{1,\,1/2,\,...,\,1/n,...\}$.

We notice that intersection of infinite set of open sets cannot be the open set. So, for instance, if $T_n=\left(-\frac{1}{n},\ 1+\frac{1}{n}\right)$, it is T_n as the interval, at any $n=1,2,3,...$ is an open set. But intersection is the segment [0, 1] that isn't an open set. For example, if it is, then the interval is an open set. But the intersection $\bigcap_{n=1}^{\infty}T_n=[0,1]$ is the interval [0, 1], which is not an open set. Because at a statement of the open sets theory, it is necessary to formulate two theorems that establish connection between the closed and open sets. In these theorems, as well as in previous ones, it is about space points sets of any number of *n*-measurements. And it should previously underlined that if *M* – the *n*-dimensional Euclidean space of points set, through *CM* is denoted the complement to the set *M*, that is a set of n-dimensional space's all points that don't belong to *M*.

7-Theorem [2]. If the set *M* is closed, then its complement *CM* is open.

Using the set theorems, we can easily discover at what cases the addition and intersection operations of sets don't bring them out from the class of open sets.

8-Theorem [2]. The sum of any set of open sets is the open set.

9-Theorem [2]. The intersection of a finite set of open sets is the open set.

A teacher has the possibility to spend more time for analysis of difficult themes and for the newest achievements in the expounded problem. Thus, a lecture transforms from a traditional lecture to the collaboration of lecture and conscious knowledge receipt, based on the already acquired student's experience.

Getting theoretical knowledge, a student has the opportunity to apply them for the solution of the tasks. A teacher directs the students' working process with the help of special questions, directing them to the independent reasoning and active search of right answer (decisions). These are the typical questions:

1. Prove that if set A\B is equivalent to the set B\A, then sets A and B are equivalent;
2. Prove the equivalence of segment and interval;
3. What is the capacity of the set of rational numbers and set of algebraic numbers?
4. What is the capacity of the set of irrational and transcendent numbers?
5. What is the capacity of the set of all polynomials with rational coefficients?
6. What is the capacity of the set of all imaginaries?

7. What is the capacity of the set of all eventual decimal fractions?
8. What is the capacity of the set of in pairs not intersecting segments is on a numerical line?
9. Will the set of rational numbers be measurable? If yes, then what is its measure of Lebesgue?
10. Why any open set and any closed set on the numerical line is measured?
11. Whether can be equal to zero measure of set that contains at least one internal point?
12. Is the function $y = \sin\sqrt{x^2+1}$ measurable on the domain of definition? If yes, then why?

13. Let $\chi(x)$ is a characteristic function of set of rational numbers. Prove that its action on any numerical function is measurable function;

14. Prove that if function f^3 is measurable, then function f is also measurable;

15. If function $|f|$ is measurable, then is it necessary for function f to be measurable?

16. Suppose that A and E are measurable sets and $A \subset E$. Calculate the Lebesgue integral $\int_E \chi_A d\mu$, where χ_A is a characteristic function of set A.

Practical employment is a basic interactive form of organization of educational process, additional to theoretical course or lecture part of educational discipline and called to help student to get used to the studied discipline and independently operate with theoretical knowledge on concrete educational material.

We will give the task for determination of coefficient of formed competence.

Task	Competence
What is the capacity of the set of all polynomials with rational coefficients?	The concept of the capacity of the set is known; Ability to apply it in a concrete situation.
Find the closure of the set of rationals.	The concept of the closure of the set is known; Ability to find the maximum points of set and its closure.
Prove the immeasurability of Dirichlet function on [0,1]	Determination of function is known; Ability to apply it in a concrete situation.
Calculate the integral from Dirichlet function on [0, 1]	The method of calculation of Lebesgue integral from everywhere broken functions is known; Ability to apply it in a concrete situation.

The change of social role of knowledge (in particular, mathematical) and creative possibilities of personality in a modern period of development of society inevitably puts questions about optimal correlation of technological and humanistic orientations in organization of educating to mathematics in pedagogical institution of higher education, conditioning for the independent mastering of new experience.

Thus, it is necessary substantially to reconstruct a structure and maintenance of mathematical preparation of students on the basis of psychology-pedagogical analysis and integration near an innovative pedagogical process taking into account experience of preceding researches.

References

1 *Iskakova A., Khanzharova B.* About another method of teaching to basic concepts of point sets theory. International Conference: Science and Education in XXI century. December 1, 2014. Bozeman, Montana, USA. — P. 117–119.
2 *Натансон И.П.* Теория действительных переменных. — М.: Наука, 1974. — С. 340.
3 *Гурвиц А., Курант Р.* Теория функций. — М.: Наука, 1968. — С. 560.

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Нақты айнымалы функциялар теориясының негіздерін оқыту туралы

Мақалада нақты айнымалы функциялар теориясы математикалық пәнді оқыту әдістерін жетілдіру мәселелері қарастырылды. Соның ішінде дәрістерді, практикалық сабақтарды жүргізу, студенттердің өзіндік жұмыстарын ұйымдастыру кезінде конструктивті түрде оқыту мәселелері. Проблемалық оқыту білім алушылардың ізденіс және шығармашылық қызмет дағдыларын қалыптастыруға мүмкіндік береді, сонымен қатар, дәстүрлі оқытуға қарағанда, көбірек уақыт қажет етсе де, оқу үрдісімен бірге білім алушының оқуға деген көзқарасына оң нәтиже береді. Авторлар теориялық материалды баяндаудың бір әдісі логикалық реттілігімен келтірілген. Практикалық сабақтардың тақырыптары ретінде дәрісте қарастырылған жалпы қорытынды жасауға мүмкіндік беретін оқыту мәселесі қойылды. Практикалық сабақтарда оқытушы білім алушыларды сұрақ-жауап арқылы олардың өз бетінше тұжырым жасауға және белсенді түрде ізденуге мүмкіндік туғызатын жұмыстарына бағыт беріп отырады. Осыған байланысты білім алушылардың құзыреттіліктерін қалыптастыру коэффициентін анықтауға мүмкіндік беретін бақылау сұрақтары келтірілді. Сонымен, біздің ойымызша, проблемалық оқыту арқылы пәнді оқып үйрену саналы білім алу түріне айналады.

Кілт сөздер: проблемалы оқыту, конструктивті оқыту, құзыреттілік, жиын өлшемдері, нақты айнымалы функциялар, теорема, оқыту, талдау.

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О преподавании основ теории функций действительного переменного

Рассмотрены вопросы совершенствования методов преподавания одной из математических дисциплин — теории функций действительного переменного. Это один из методов активного обучения, способствующий организации поисковой деятельности обучающихся, формированию навыков творческого изучения дисциплины. Отмечено, что проблемное обучение положительно сказывается как на процессе обучения, так и на отношении обучаемого к самому процессу, хотя и занимает более продолжительное время при подаче нового материала, нежели традиционный метод. В статье приведены один из методов изложения теоретического материала, основные понятия, утверждения в их логической последовательности. Для практического занятия в качестве темы выбрана такая учебная задача, которая предполагает потребность обучающегося «потрогать» материал, опознать в конкретном то общее, о чем говорилось на лекции. Показано, что на практических занятиях преподаватель направляет работу обучающихся с помощью вопросов, побуждающих их к самостоятельному рассуждению и активному поиску правильного решения. В связи с этим приведены контрольные вопросы, а также задания для определения коэффициента сформированности компетенции обучающегося. Таким образом, авторы считают, что изучение дисциплины превращается из традиционной формы обучения в сотрудничество и сознательное приобретение знаний.

Ключевые слова: проблемное обучение, компетенция, интерактивная форма обучения, функция действительных переменных, измерение множеств, теорема, анализ.

References

1 Iskakova A., Khanzharova B. *About another method of teaching to basic concepts of point sets theory*. International Conference: Science and Education in XXI century. December 1, 2014, Bozeman, Montana, USA. — p. 117–119.

2 Natanson I.P. *The theory of real variables*, Moscow: Nauka, 1974, p. 340.

3 Hurwitz A., Courant R. *The theory of functions*, Moscow .: Nauka, 1968, p. 560.